DESIGN RESEARCH ON MATHEMATICS EDUCATION: INVESTIGATING THE PROGRESS OF INDONESIAN FIFTH GRADE STUDENTS' LEARNING ON MULTIPLICATION OF FRACTIONS WITH NATURAL NUMBERS

A THESIS

Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science (M.Sc)
in
International Master Program on Mathematics Education (IMPoME)
Graduate School Sriwijaya University
(In Collaboration between Sriwijaya University and Utrecht University)

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GRADUATE SCHOOL
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APPROVAL PAGE

Research Title : Design Research on Mathematics Education: Investigating the Progress of Indonesian Fifth Grade Students' Learning on Multiplication of Fractions with Natural Numbers

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State that:

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ABSTRACT

This study aimed at investigating the development of students' learning multiplication fractions with natural numbers through different levels. Therefore, design research was chosen to achieve this research goal and to contribute in formulating and developing a local instruction theory for teaching and learning of multiplication fractions with natural numbers. In design research, the Hypothetical Learning Trajectory (HLT) plays important role as a design and research instrument. It was designed in the phase of preliminary design and tested to thirty-seven students (i.e., five students in pilot experiment and thirty-two students in teaching experiment) of grade five primary school in Indonesia (i.e. SDN 179 Palembang which had been involved in Pendidikan Matematika Realistik (PMRI) or Indonesian Realistic Mathematics Education project since 2009). The designed HLT was then compared with students' actual learning in the experimental phase and analyzed in which students learned or did not learn from what we had conjectured them to learn in the retrospective analysis phase. The result of the experiments showed that *length measurement* activity could stimulate students' informal knowledge of *partitioning* in order to produce fractions as the first level in learning multiplication of fractions with natural numbers. Furthermore, strategies and tools used by the students in partitioning gradually be developed into a more formal mathematics in which the representation of yarn be used as the *model of* measuring situation. The representation of yarn which then called the number line could bring the students to the last activity levels, namely on the way to rules for multiplying fractions with natural numbers, and became the *model for* more formal reasoning. Based on these findings, it can be concluded that students' learning multiplication of fractions with natural numbers in which the learning process become a more progressive learning developed through different levels.

Key Words: multiplication fractions with natural numbers, length measurement activity, design research, Hypothetical Learning Trajectory, *model of*, *model for*.

ABSTRAK

Penelitian ini bertujuan untuk mengkaji kemajuan belajar siswa pada materi perkalian pecahan dengan bilangan bulat melalui berbagai tingkatan yang berbeda-beda. Oleh karena itu, *design research* dipilih untuk mencapai tujuan penelitian dan untuk berkontribusi dalam merumuskan dan mengembangkan teori instruksi lokal dalam pembelajaran perkalian pecahan dengan bilangan bulat. Dalam *design research*, lintasan belajar (*Hypothetical Learning Trajectory*) memegang peranan penting sebagai desain dan instrumen penelitian. Lintasan belajar dirancang dalam tahap desain awal dan diujikan pada 37 siswa (yaitu, 5 siswa pada percobaan kelompok kecil dan 32 siswa pada percobaan kelas) kelas lima di Indonesia (yaitu, SDN 179 Palembang yang telah terlibat dalam proyek Pendidikan Matematika Realistik Indonesia (PMRI) sejak tahun 2009). Lintasan belajar yang telah dirancang kemudian dibandingkan dengan pembelajaran siswa sebenarnya pada tahap percobaan dan dianalisis dimana siswa belajar atau tidak belajar dari apa yang telah kita duga mereka untuk belajar pada tahap analisis retrospektif. Hasil penelitian menunjukkan bahwa aktivitas pengukuran panjang dapat menstimulasi pengetahuan informal siswa dalam mempartisi untuk menghasilkan pecahan sebagai level pertama dalam tahapan pembelajaran perkalian pecahan dengan bilangan bulat. Selanjutnya, strategi-strategi dan alat yang digunakan oleh siswa dalam mempartisi secara bertahap dikembangkan menjadi matematika yang lebih formal dimana garis bilangan digunakan sebagai *model of* situasi pengukuran. Representasi dari benang yang kemudian disebut garis bilangan dapat membawa siswa menuju tingkat aktivitas akhir, yaitu dalam perjalanan menuju aturan perkalian pecahan dengan bilangan bulat, dan menjadi *model for* penalaran yang lebih formal. Berdasarkan temuan-temuan yang didapatkan, dapat disimpulkan bahwa pembelajaran siswa mengenai materi perkalian pecahan dengan bilangan bulat dimana proses belajar lebih progresif berkembang melalui tingkatan yang berbeda-beda.

Kata Kunci: perkalian pecahan dengan bilangan bulat, aktivitas pengukuran panjang, *design research*, lintasan belajar, *model of*, *model for*,

SUMMARY

The domain of skill and knowledge referred to as "fractions" has been parsed in various ways by researchers in recent years. Focus on this study, an important developmental step in learning multiplication of fractions with natural numbers is the transition of counting principles from multiplying natural numbers to multiplying fractions. To scaffold this transition, we tried to design a sequence of activities in which length measurement activity be used as a starting point in teaching and learning process.

This study aimed at investigating the development of students' learning of multiplication fractions with natural numbers through different levels. In the design research, the Hypothetical Learning Trajectory (HLT) served as a design and research instrument. This HLT was tested to thirty-seven students of fifth grade (i.e. SDN 179 Palembang which had been involved in the Pendidikan Matematika Realistik (PMRI) or Indonesian Realistic Mathematics Education project since 2009) through two cycles, namely pilot and teaching experiments. The pilot experiment was conducted in the first two weeks to five students, excluded from teaching experiment class. Based on the observations in the pilot experiment, initial HLT was then improved and adjusted to be used in the real teaching experiment.

The sequence of activities in our HLT was designed based on levels which preceded the learning operation of fractions. Therefore, in this summary, we will describe a brief result of our research based on those levels.

1. **Producing Fractions**
   To produce fractions, the students worked on the first and the second activity. Through '*locating eight flags and six water posts*' activity, it was found that length measurement activity could bring the students to perceive the idea of *partitioning* a certain length into some parts equally. The activities was then continued by notating fractions for each part of the partitioning result. In this activity, students' knowledge about the meaning of fractions as part of a whole was required. As a result, the student could produce fractions, namely unit (i.e., $\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8}$ and $\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6}$) and non-unit fractions (i.e., $\frac{1}{8},\frac{2}{8},\frac{3}{8},\frac{4}{8},\frac{5}{8},\frac{6}{8},\frac{7}{8},\frac{8}{8}$ and $\frac{1}{6},\frac{2}{6},\frac{3}{6},\frac{4}{6},\frac{5}{6},\frac{6}{6}$).
2. **Generating Equivalencies**
   In the second activity, the students were also asked to describe the relation among fractions (e.g the relation between $\frac{1}{8}$-jumps with point $\frac{5}{8}$). In this activity, the students came to the drawing a representation of yarn used to measure the length of running route as a *model of* measuring situation. This representation was then called number line of fractions in which fractions appeared on the number line. Through the number line, students' knowledge about equivalent fractions which can be seen from fractions on the same position was explored well. The students could also grasp the idea of '*jumps*' which then led them to the idea of multiplication of natural numbers by fractions as repeated addition of fractions. They found out that there were five times $\frac{1}{8}$-jumps to reach point $\frac{5}{8}$ from zero point. They then came up with the idea that $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ is the same as $5\times\frac{1}{8}$.

3. **Operating through Mediating Quantity**
By attaching the length of the route in the activity of *'determining who is running farther'*, the students were able to find $\frac{5}{8}$ of 6 kilometers and $\frac{4}{6}$ of 6 kilometers. The length of running route served as a mediator to lead the idea of fractions as operator. In this activity, double number line model appeared as a model connected to the activity of partitioning the yarn (activity 1). Besides, there was a surprising idea from some students in working group session who could find the answer of $\frac{4}{6}$ of 6 kilometers by only multiplying the length with the numerator of fraction then put the denominator bellow the result. They came up with the rule of operation with fraction when they saw the connection between $5\times\frac{1}{8}$ with $\frac{5}{8}$.
4. **Doing One's Own Production**
At this level, progression meant that the students were able to solve problems in a more and more refined manner at the symbolic level. As mentioned in the third tenet of RME, the biggest contributions to the learning process were coming from *student's own creations and contributions* which led them from their own informal to the more standard formal methods. The group who found the rule of operation with fraction had made a significant contribution to the classroom community for the idea of multiplying fractions by natural numbers. However, there were some students who still used the double number line to solve the problem of multiplication fractions by natural number. It means that the double number line model still made sense for those students.
5. **On the Way to Rules for the Operations with Fractions**
In the formal level, students' reasoning with conventional symbolizations started to be independent from the support of models for mathematical activity. The transition to a more formal operation with fractions was preceded by stimulating students to contribute their own informal ways of working which led by the group who found the rule of multiplying fractions by natural numbers. At this level, it was shown that students' contribution became *model for* more formal reasoning and the *general level* of modeling had been attained by the students.

Based on the findings, it can be concluded that the use of length measurement context has stimulated students to start the first activity level in learning multiplication of fractions with natural numbers. It was started from producing their own fractions until on the way to rules for multiplying fractions with natural numbers. The emergence of model supports students' transition from concrete situational problems to a more formal and general mathematics. Through number line as a *model of* measuring situation, the students came up with the idea of multiplication of natural numbers by fractions as repeated addition of fractions. The number line then developed into a more complex number line in which a natural number attached on the number line which then later called as double number line. Through the double number line, the students could construct their knowledge on the way to rules in multiplying fractions by natural numbers as a *model for* more formal reasoning.

RINGKASAN

Domain dari keterampilan dan pengetahuan tentang “pecahan” telah dipaparkan dengan berbagai cara oleh para peneliti dalam beberapa tahun terakhir. Fokus pada studi ini, langkah penting dalam perkembangan belajar perkalian pecahan dengan bilangan bulat adalah pada masa transisi prinsip perhitungan dari mengalikan bilangan bulat ke mengalikan pecahan. Untuk menjembatani masa transisi ini, kami mencoba untuk merancang suatu urutan kegiatan dimana aktivitas pengukuran panjang digunakan sebagai titik awal dalam proses belajar mengajar.

Penelitian ini bertujuan untuk mengkaji kemajuan belajar siswa pada materi perkalian pecahan dengan bilangan bulat melalui tingkatan yang berbeda-beda. Dalam *design research*, lintasan belajar (*Hypothetical Learning Trajectory*) berfungsi sebagai desain dan instrumen penelitian. Lintasan belajar ini kemudian diujicobakan pada 37 siswa kelas lima (yaitu, SDN 179 Palembang) melalui dua siklus, yaitu percobaan kelompok kecil dan percobaan kelas. Percobaan kelompok kecil dilakukan dalam dua minggu pertama pada lima siswa dimana siswa tersebut tidak termasuk dalam percobaan kelas. Berdasarkan pengamatan dalam percobaan kelompok kecil, lintasan belajar awal kemudian diperbaiki dan disesuaikan untuk kemudian digunakan dalam percobaan kelas.

Urutan kegiatan belajar dirancang berdasarkan tingkatan-tingkatan dalam membelajarkan operasi hitung pada pecahan. Oleh karena itu, dalam ringkasan ini, kami akan menjelaskan secara singkat hasil penelitian kami berdasarkan tingkatan-tingkatan tersebut.

1. **Menghasilkan Pecahan**

   Untuk menghasilkan pecahan, siswa mengerjakan aktivitas satu dan dua. Melalui aktivitas ‘menempatkan delapan bendera dan enam stan air minum’, ditemukan bahwa aktivitas pengukuran panjang dapat menggiring siswa menuju ide mempartisi suatu panjang tertentu menjadi beberapa bagian sama panjang. Kegiatan kemudian dilanjutkan dengan menuliskan bentuk pecahan untuk setiap bagian hasil partisi. Dalam aktivitas ini, pengetahuan siswa mengenai arti pecahan sebagai bagian dari keseluruhan diperlukan. Hasilnya, siswa dapat menghasilkan pecahan, yaitu unit pecahan ($\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8}$ dan $\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6}$) dan non-unit pecahan ($\frac{1}{8},\frac{2}{8},\frac{3}{8},\frac{4}{8},\frac{5}{8},\frac{6}{8},\frac{7}{8},\frac{8}{8}$ dan $\frac{1}{6},\frac{2}{6},\frac{3}{6},\frac{4}{6},\frac{5}{6},\frac{6}{6}$).

2. **Menghasilkan Kesamaan**

   Pada aktivitas kedua, siswa juga diminta untuk menggambarkan hubungan antara pecahan (sebagai contoh, hubungan antara lompatan $\frac{1}{8}$ dengan titik $\frac{5}{8}$). Pada aktivitas ini, siswa menggambarkan representasi dari benang yang digunakan untuk mengukur panjang lintasan lari sebagai model dari situasi pengukuran. Representasi ini kemudian disebut garis bilangan pecahan dimana pecahan-pecahan muncul di garis bilangan. Melalui garis bilangan, pengetahuan siswa mengenai pecahan senilai yang dapat dilihat dari pecahan-pecahan yang berada pada posisi yang sama dieksplorasi dengan baik. Siswa juga dapat memahami ide tentang lompatan pada garis bilangan pecahan yang kemudian menggiring mereka menuju ide perkalian bilangan bulat dengan pecahan sebagai penjumlahan berulang terhadap pecahan. Mereka menemukan bahwa ada lima

kali lompatan $\frac{1}{8}$ untuk mencapai titik $\frac{5}{8}$ dari titik nol. Mereka kemudian datang menuju ide bahwa $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ sama dengan $5\times\frac{1}{8}$.

3. **Melakukan Operasi Hitung melalui Mediasi Kuantitas**
   Dengan melibatkan panjang lintasan lari pada aktivitas “menentukan siapa yang lari lebih jauh”, siswa mampu menemukan jawaban $\frac{5}{8}$ dari 6 kilometer dan $\frac{4}{6}$ dari 6 kilometers. Panjang lintasan lari berfungsi sebagai perantara untuk menuju ke ide pecahan sebagai operator. Pada aktivitas ini, garis bilangan ganda muncul sebagai sebuah model berkaitan dengan aktivitas mempartisi benang (aktivitas 1). Selain itu, ditemukan sebuah ide dari sekelompok siswa yang dapat menemukan jawaban $\frac{4}{6}$ dari 6 kilometer dengan hanya mengalikan panjang lintasan lari dengan pembilang pecahan dan kemudian menyimpan penyebut pecahan dibawah hasil perkalian tersebut. Mereka memiliki ide aturan operasi pada pecahan ketika melihat hubungan $5\times\frac{1}{8}$ dengan $\frac{5}{8}$.
4. **Melakukan Produksi Sendiri**
   Pada level ini, suatu perkembangan diartikan ketika siswa mampu memecahkan masalah dengan cara yang lebih ke arah simbolis. Seperti disebutkan pada prinsip ketiga pendekatan *RME (Realistic Mathematics Education)*, kontribusi terbesar dalam proses belajar berasal dari kreasi dan kontribusi siswa sendiri yang kemudian menggiring mereka dari informal menuju ke arah yang lebih formal. Kelompok yang menemukan aturan operasi pada pecahan memberikan kontribusi yang sangat signifikan pada komunitas kelas mengenai ide perkalian pecahan dengan bilangan bulat. Namun, ada beberapa siswa yang masih menggunakan garis bilangan ganda untuk menyelesaikan masalah berkaitan dengan perkalian pecahan dengan bilangan bulat. Hal ini dimungkinkan karena penggunaan garis bilangan ganda masih melekat pada siswa tersebut.
5. **Dalam Perjalanan Menuju Aturan Operasi pada Pecahan**
   Pada tingkat formal, penalaran siswa menggunakan simbol konvensional mulai lepas dari dukungan model untuk aktivitas matematika. Transisi untuk ke arah operasi yang lebih formal diawali dengan menstimulasi siswa untuk berkontribusi pada cara bekerja informal mereka sendiri yang kemudian menggiring suatu kelompok untuk menemukan aturan perkalian pecahan dengan bilangan bulat. Level ini menunjukkan bahwa kontribusi siswa menjadi model untuk penalaran yang lebih formal dan tingkat umum pemodelan telah dicapai oleh siswa.

Berdasarkan temuan-temuan tersebut dapat disimpulkan bahwa penggunaan konteks pengukuran panjang telah menstimulasi siswa untuk memulai level aktivitas pertama dalam belajar perkalian pecahan dengan bilangan bulat. Hal ini dimulai dari menghasilkan pecahan sendiri hingga menuju ke aturan perkalian pecahan dengan bilangan bulat. Munculnya model mendukung masa transisi siswa dari masalah situasi yang nyata menuju ke arah matematika yang lebih formal dan lebih umum. Melalui garis bilangan sebagai model dari situasi pengukuran, siswa dapat memunculkan ide perkalian bilangan bulat dengan pecahan sebagai penjumlahan pecahan berulang. Garis bilangan ini kemudian berkembang menjadi garis bilangan yang lebih kompleks yang kemudian dinamakan garis bilangan ganda. Melalui garis bilangan ganda, siswa dapat mengkonstruksi pengetahuan mereka menuju ke aturan perkalian pecahan dengan bilangan bulat sebagai model penalaran formal.

PREFACE

Experience is not what happens to you. It is what you do with what happens to you. Many people contributed to my development. Therefore, to what is set down in this thesis, I thank them all for helping me to write this thesis; supervisors, colleagues, teachers and students who involved in this research, and many others.

Experiments reported in this thesis were conducted at SDN (Sekolah Dasar Negeri/*State Primary School*) 179 Palembang which had been involved in the Pendidikan Matematika Realistik (PMRI) or Indonesian Realistic Mathematics Education project since 2009. This school offered me the chance to work with their students. The school's headmistress and teachers helped me in many ways to conduct the study reported upon here. I owe them my gratitude for the flexibility they expressed in sharing their school. I especially thank Dra. Yuliani, M.M., the school's headmistress, and Mrs. Sri Horestiati, teacher who involved in this research, for their support.

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CHAPTER 1
INTRODUCTION

Researches have identified major problems with current learning methods for teaching fractions. The first dealt with a syntactic (rules) rather than semantic (meaning) emphasis of learning rational numbers, where the learning processes often emphasize technical procedures in doing fraction operations at the expense of developing a strong sense in students of the meaning of rational numbers (Moss & Case, 1999). This problem leads to algorithmically-based mistakes, which result when an algorithm is viewed as a meaningless series of steps so that students often forget some of these steps or change them in ways that lead to errors (Freiman & Volkov, 2004).

In learning multiplication by fractions in Indonesia, most of the students are required to master the procedures and algorithms. They just need to memorize formulas and tricks in calculation to solve the problems. However, we do not know whether the students know and understand the meaning of the procedures and algorithms lay behind it. Even we ourselves never fully understand why we need to multiply the numerator and the denominator together to multiply two fractions until now. As stated in P4TK (Pusat Pengembangan Pemberdayaan Pendidik dan Tenaga Kependidikan), teachers who play a crucial role in the mathematical learning outcomes of their students still have weaknesses in mastering the material, the preparation, use of media, as well as the selection of strategies/methods in teaching topic about multiplication by fractions (P4TK, 2009). Most students know how to apply the procedure without understanding why the procedure works (de Castro, 2004).

Secondly, the learning processes often takes an adult-centered rather than a child-centered approach, emphasizing a fully formed adult conception of rational numbers, not taking into consideration schema and informal knowledge of fractions, thus denying students a spontaneous means of learning fractions (Moss & Case, 1999). One of the reasons points out as to why the mathematical notion of fractions is systematically misinterpreted because fractions are not consistent with the counting principles that apply to natural numbers to which students often relate (Stafylidou & Vosniadou, 2004). Focus on the multiplication in counting principles, in multiplying natural numbers, the product is larger than the factor. On the other hand, in multiplying fractions, the product may either be higher or lower than the factors. The fact that multiplication by fractions does not increase the value of the product might confuse those who remember the definition of multiplication presented earlier for natural numbers.

Considering the two aforementioned issues, it seems to be necessary to remodel mathematics teaching and learning, especially in domain multiplication fractions with natural numbers. Therefore, we conducted a design research that developed a sequence of activities emphasizing on a shift from “mastering procedures” to “understanding”. In proposing this design research we referred to Realistic Mathematics Education (RME) that has been developed in the Netherlands since in early 1970s. This method is based on the idea of mathematics as a human activity and as a constructive activity.

According to Streefland (1991), there are five levels in learning operation with fractions, namely producing fractions, generating equivalencies, operating through mediating quantity, doing one’s own productions and on the way to rules for the operations with fractions. In this study, we follow Streefland by promoting problem on the context at any stage. We proposed the use length measurement as the contextual situation of the activities to support students’ learning. The context of running route was used as a starting point to introduce the number line which could be used as a helpful tool to solve problems related to fractions and natural numbers. The aim of the research was to investigate the development of students' learning of multiplication fractions with natural numbers through different levels. Therefore, the research question was formulated as follows.

"How does students' learning of multiplication fractions with natural numbers develop through different levels?"

In this study, we tried to focus the issues concerning the development of students' learning of multiplication fractions with natural numbers through different levels. Therefore, we attempted to answer sub-research questions:

1. *How does the length measurement activity with the help of yarn provoke students in producing their own fractions?*
2. *How does the string of yarn lead the students to the idea of number line?*
3. *How does a number line lead the students to the idea of multiplication of natural numbers by fractions by generating equivalencies?*
4. *How does the involvement of certain length as the mediating quantity lead the students to the idea of multiplication of fractions by natural numbers?*
5. *How does student's own production about the idea of multiplication of fractions lead them on the way to rules for multiplying fractions by natural numbers?*

CHAPTER 2
THEORETICAL FRAMEWORK

This chapter provides the framework of thinking used in this study. Literatures about fraction as magnitude and multiplication by fractions were studied to design the instructional activities of multiplication fractions with natural numbers.

In this research, length measurement activity was used as a starting point and contextual situations to investigate the development of students' learning multiplication fractions with natural numbers. To explain and investigate how the framework of learning multiplication of fractions with natural numbers in the measurement activity bring the students to the more formal mathematics, we also discussed about the Realistic Mathematics Education approach.

Because the research conducted in Indonesia, this chapter also provides an overview about multiplication by fractions for elementary school in Indonesian curriculum.

A. Fraction as Magnitude

Early fractions instruction generally focuses on the idea that fractions represent parts of a whole (e.g., one-third as the relation of one part to a whole that has three equal parts). Although the part-whole interpretation of fractions is important, too often instructions do not convey another simple but critical idea: fractions are numbers with magnitudes (values).

In the mathematics point of view, magnitude means the quantity of the objects. Freudenthal (1983) defined magnitude as the mathematical model that fits the mathematical distribution problem. For instance, the mathematical distribution problem is about the distribution of three breads for four people. Here, the quantity replaces each other in order to be considered equal. Moreover, Freudenthal also stated that in phenomenological approach we must start with the objects in which equivalence relation can form classes representing the magnitude of values.

Freudenthal also defined four requirements to describe the magnitude in a system of quantities. We selected two of the requirements which relate to multiplication by fractions, namely:

1. An equivalent relation which describes the conditions for replacing objects. For instance, quantities of a certain substance, with each other and which lead to the *equality* within the magnitude.
2. The possibility of dividing an object into an arbitrary number of partial objects that replaced each other, which led to *division by natural number*.

Connected with multiplication by natural number, Freudenthal (1983) defined multiplication by natural numbers as a repeated addition: "*n*th part of" becomes the inverse of "*n* times". By composing multiplications and divisions with each other we came to the mathematical idea of *multiplications by fractions*.

B. Multiplication by Fractions

Streefland (1991) suggested five activity levels in learning operation with fractions, namely producing fractions, generating equivalencies, operating through mediating quantity, doing one's own productions and on the way to rules for the operations with fractions. In this sub chapter, we focused on the five activity levels that are offered by Streefland, more specific to our main domain which is multiplication by fractions. The levels are described as follows.

1. Producing fractions.

The activities here are concentrated in providing contexts at the concrete level. In order to solve all the problems, fractions material is produced by means of estimation and varied distribution. Divergent contexts and processes are explored which could produce fractions, such as fair sharing, division, length measurement, making mixtures, combining and applying recipes. By linking to the involved magnitudes and varying the objects, the notion of a fraction in operator became clear. For instance, a certain condition, when applies to partition a certain length, will naturally cause a variety of solutions with accompanying notations of fractions.

2. Generating equivalencies.

After students has experienced with notating fractions of their own fractions production, the learning process will continue to generate equivalencies as students are asked to determine fractions in the same position. Equivalencies occur when the distribution problem is given, for instance, the case of partitioning a certain length into eight parts. The equivalencies may occur when students were asked about the relation between $\frac{5}{8}$ and $\frac{1}{8}$.

The multiplicative reasoning of fractions within fractions equivalency also refers to two definitions, namely fractions involve *between* – and *within – multiplicative relations* (Vanhille & Baroody, 2002). Between-multiplicative relation refers to the relation between the numerators and the denominators of equivalent fractions. If two fractions are equivalent, the ratio between the numerators is the same as the ratio between the denominators. Within-multiplicative relation refers to the relation between the numerator and the denominator of a fraction. For example, in $\frac{3}{4}$ the numerator, 3, is $\frac{3}{4}$ of the denominator 4 (i.e., $3=\frac{3}{4}\times 4$); and the denominator, 4, is $\frac{4}{3}$ of 3 (i.e., $4=\frac{4}{3}\times 3$). All equivalent fractions share the same within-fraction multiplicative relation.

3. Operating through mediating quantity.

To lead to the idea of fractions as operator, we can involve the length to a given unit. The fraction which at the first is described as part of a whole relationship now become a fraction *in* an operator. Based on Fosnot and Dolk (2002), this concept is important because it will connect to the idea of double number line. Taber (1992) suggested that instruction of multiplication with fractions shall relate to multiplication with natural numbers while reconceptualizing students' understanding of natural numbers multiplication to include fractions as multipliers.

4. Doing one's own productions.

At this moment, we cannot put high expectation that the students will come up with their own production. Therefore, questions which can provoke them are needed at this level. Multiplication strategies for fractions can be built upon this equivalence. For instance, the decompositions of $\frac{5}{8}$, can be carried out in $\underset{8}{\textcircled{5}}$ with the aid of unit by unit division, which is in the meantime becoming a more standard procedure: $\frac{5}{8} = 5 \times \frac{1}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$. At this moment, students can also explore multiplication fractions as repeated addition. Connected with the third level, when natural number involves in the multiplication fractions, the use of fractions as multipliers is important. For instance, when students are asked to find five-eighth of 6 kilometers which can be written in more formal mathematics as $\frac{5}{8} \times 6$. At this level, progression means that the students are able to solve problems in a more and more refined manner at the symbolic level.

5. On the way to rules for the operations with fractions.

Within mini lesson which include fractions as multipliers, the students reflect on the rules for the multiplication by fractions operations which may be in force here. The transition to more formal fractions is preceded by stimulating students to contribute their own informal ways of working. Therefore, in the mini lesson, it is possible to bring about this transition to application of more formal rules.

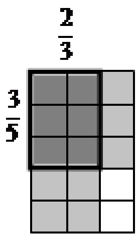
The five activity levels above are built upon a set of big ideas and strategies underlying multiplication with fractions which are described in the following table.

Table 1. The Big Ideas of Multiplication with Fractions that are Formulated by Fosnot & Hellman (2007)

Big Ideas	Description
Fractions represent a relation.	Fractions are relations: a ratio of part of whole (five parts out of eight).
The whole matter.	Fractions can also be used as operators. For instance, consider the operation of multiplication: $\frac{3}{8}$ of 1 kilometers. Here $\frac{3}{8}$ refers to one unit measurement broken into eighths with three of them marking $\frac{3}{8}$ of the unit ($3\times\frac{1}{8}$). It is also possible to think of $\frac{3}{8}$ of $\frac{1}{2}$ kilometers as $\frac{3}{16}$ of 1 kilometers. To determine what to name the fraction in such cases we must refer to the whole, it matters.
To maintain equivalence, the ratio of the related number must be kept constant.	In traditional way, students have been taught to make equivalent fractions by multiplying of dividing by one (in the form of $\frac{2}{2}$, $\frac{3}{3}$, etc). But when they need to give argument that $\frac{3}{6}=\frac{4}{8}$, understanding the implicit ratio is required. If two fractions are equivalent, the ratio between the numerators is the same as the ratio between the denominators.
The properties (distributive, associative and commutative) that hold for natural numbers also hold for rational numbers.	Students may have worked on these properties with natural number operations. However, when they move to fractions, they are often surprise to see that $\frac{1}{8}\times 5$ is equivalent to $5\times\frac{1}{8}$ even in the context it is somewhat different. They may not think to use partial product like $\frac{1}{2}$ (or $\frac{4}{8}$) of 6 plus $\frac{1}{8}$ of 6 to calculate $\frac{5}{8}$ of 6 kilometer. Students will probably need to spend time exploring these properties with rational numbers in order to generalize the use.

As students work with the five activity levels, we notice that students may use many strategies for multiplication by fractions. The strategies to notice are described in the following table.

Table 2. The Strategies of Multiplication with Fractions that are Formulated by Fosnot & Hellman (2007)

Strategies	Description
Using repeated addition to find a fraction of a whole.	Students often begin to use repeated addition strategies. For instance, to find $\frac{5}{8}$ of 6 kilometers, they will find $\frac{1}{8}$ by dividing 6 into 8 equal parts, then they will add these altogether: $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}$.
Using multiplication and division to make equivalent relations.	When students begin to realize that $\frac{5}{8}$ means by five times of $\frac{1}{8}$, they begin to use this strategy: they divide the number by the denominator and multiply by the numerator.
Using landmark fractions to make partial products.	Calculating $\frac{5}{8}$ of 6 kilometer can be simple if students think about to use landmark fractions. A half of 6 is 3, then the students only need to think $\frac{1}{8}$ of 6 and add the two partial products together. The distributive property for multiplication over addition underlies this strategy.
Using a ratio table as a tool for making equivalent fractions.	Ratio table is a powerful strategy and should be encouraged. Figuring out $\frac{1}{8}$ of 6 by using $\frac{1}{2}$ of 6 and halving it ($\frac{1}{4}$ of 6 = $\frac{1}{2}$ of 3) and halving again ($\frac{1}{8}$ of 6 = $\frac{1}{2}$ of 1½ = ¾) is evidence proportional reasoning and an understanding of the relationship involved.
Using the standard algorithm for multiplication of fractions.	Multiply the numerators and the denominators together. This standard algorithm can be quickly understood when the array model is used. For instance, $\frac{3}{5}\times\frac{2}{3}$ is shown in the model below. The product is $\frac{6}{15}$. The outer rectangle is formed by the denominators (5×3), and the inner rectangle is formed by the numerators (3×2). 

The use of array model as a strategy to solve multiplication by fractions which is formulated by Fosnot & Hellman above can be used as a helpful tool to teach multiplication of fractions by fractions. However, this study only focused on multiplication fractions with natural numbers. Therefore, we did not discuss the use of array model at this present study.

C. Realistic Mathematics Education (RME)

Realistic Mathematics Education (RME) is a theory of mathematics education which has been developed in the Netherlands since 1970s. This theory is strongly influenced by Hans Freudenthal's concept of 'mathematics as a human activity' (Freudenthal, 1991). According to Freudenthal, students should not be treated as passive recipients of ready-made mathematics, but rather than education should guide the students towards using opportunities to discover and reinvent mathematics by doing it themselves. Therefore, this study developed a sequence of instructional activities on multiplication fractions with natural numbers in which the students understand the problem instead of mastering the procedures and algorithms. To shift from the situational activities to the more formal mathematics, the tenets of Realistic Mathematics Educations (RME) offered clues and design heuristics.

1. Five Tenets of RME

The process of designing a sequence of instructional activities that started with contextual situation in this study is inspired by five tenets in Realistic Mathematics Education as a combination of Van Hiele's three levels, Freudenthal's didactical phenomenology and Treffer's progressive mathematization (Treffers, 1991). The descriptions are as follows.

1. *The use of contextual problems.*

Contextual problems figure as applications and as starting points from which the intended mathematics can come out. The mathematical activity is not started from a formal level as students usually face with, but from a situation that is experientially real for students. Consequently, this study used the running race route as the context in which the students could act and reason to the given problems.

2. *The use of models or bridging by vertical instruments.*

Broad attention is paid to the development models, schemas and symbolizations rather than being offered the rule or formal mathematics right away. Students' informal knowledge as a result of students' experience in making partitioning using tools (i.e., yarn) needed to be developed into formal knowledge of fractions which would lead to the idea of equivalent fractions when the notation of the result of partitioning were put on the string of yarn. The use of string of yarn here was as a bridge to the number line model which was in more abstract level.

3. *The use of students' own creations and contributions.*

The biggest contributions to the learning process are coming from student's own constructions which lead them from their own informal to the more standard formal methods. Students' strategies and solutions can be used to develop the next learning process. The use of string of yarn served as the base of the emergence model of number line.

4. *The interactive character of the teaching process or interactivity.*

The explicit negotiation, intervention, discussion, cooperation and evaluation among students and teachers are essential elements in a constructive learning process in which the students' informal strategies are used to attain the formal ones. Through discussions about running race problems in each day which were designed in continuity story, students could communicate their works and thoughts in the social interaction emerging in the classroom.

5. The intertwining of various mathematics strands or units.

From the beginning of the learning process, the learning activities of fractions are intertwined with proportion. This means that explanation of the unifying relationship between, for instance, equivalent fractions and proportion was not kept until the very end of the learning process. Moreover, learning multiplication by fractions through length measurement activity would also support the development of students' skills and ability in the domain of geometry.

2. Emergent Modeling

Gravemeijer suggested that instead of trying to help students to make connections with ready-made mathematics, we should try to help students construe mathematics in a more bottom-up manner (Gravemeijer, 1999, 2004). This recommendation fits with the idea of emergent modeling. Modeling in this conception is an activity of the students.

Initially, the models refer to concrete situations, which are experientially real for the students. When the students have more experience with similar problems, their attention may shift towards the mathematical relations and strategies. As a consequence, the model becomes more important as a base for mathematical reasoning than as a way to solve the problem. In this situation, the model started to become a referential base for the level of formal mathematics. Or in short, a *model of* informal mathematical activity develops into a *model for* more formal mathematical reasoning.

Gravemeijer elaborated the *model of* and *model for* distinction by identifying four general types of activity (Gravemeijer, 1994), as shown in the following figure 1.

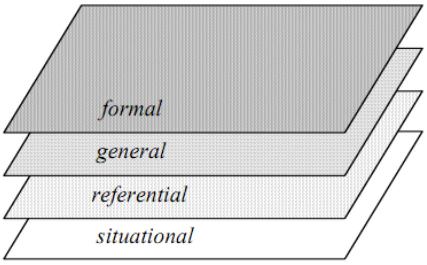


Figure 1. Levels of mathematical activity

The implementation of the four general types of activities in this study is described as follows.

(1) *Situational activity*, in which interpretations and solutions depend on the understanding of how to act and to reason in the setting. The setting in this present study was the context of running race route. In this level, through the problem of "*locating flags and water posts on the running route*", students would explore their informal knowledge of partitioning when they were asked to divide a certain length into some equal parts.

(2) *Referential activity*, in which *model of* refer to activity in the setting described in instructional activities. Students' activities might be considered referential when they were initially use tools (yarn) as a representation of running race route. In this study, the activity of "*notating fractions in the empty fractions cards and putting fractions cards on the string of yarn*" also served as referential activity in which students produced their own fractions to represent their way in making construction of partitioning. In this activity, the representation of string of yarn became the *model of* measuring situation.

(3) *General activity*, in which *model for* refer to a framework of mathematical relations. The *model for* more mathematical reasoning in this present study was the number line. In this activity, number line was introduced as a generalization tool of string of yarn.

Students were asked to describe the relation among fractions which they could see from their own fractions production. In addition, the activity of "*determining who is running farther*" which the length of the running race route was involved would lead students to the idea of fractions as operator. At this moment, the double number line could be used as a helpful tool to find the distance that could be covered by two runners where their locations were known with the help of flags and water posts.

(4) *Formal mathematical reasoning* which is no longer dependent on the support of *models for* mathematical activity. The focus of the discussion moves to more specific characteristics of models related to the concepts of equivalent fractions, fractions as operator and fractions as multipliers.

D. Multiplication by Fractions in Indonesian Curriculum for Primary School

Topic about fractions in Indonesia has been taught since in the third grade in which students learn about the meaning of fractions and comparing simple fractions. In the fourth grade, students begin to learn operation in fractions, namely addition and subtraction of fractions. Table 3 describes Indonesian curriculum of fractions in the second semester of grade five.

Table 3. Fractions for Primary School Grade Five in the Second Semester in Indonesian Curriculum.

Standard Competence	Basic Competence
Number 5. Using fractions in solving problems.	5.1 Changing fractions to percentages and decimals and vice versa. 5.2 Adding and subtracting fractions. 5.3 Multiplying and dividing fractions. 5.4 Using fractions in solving problems involving ratio and scale.

Based on the curriculum above, we conducted one of the basic competences, namely multiplying fractions. In more specific domain, we focused on multiplication fractions with natural numbers.

CHAPTER 3
RESEARCH METHODOLOGY

A. Design Research

The type of research that we used was design research (Gravemeijer & Cobb, 2001) that also termed as developmental research because instructional materials were developed.

> "Developmental research means: "experiencing the cyclic process of development and research so consciously, and reporting on it so candidly that it justifies itself, and that this experience can be transmitted to others to become like their own experience."
> (Freudenthal, 1991, p. 161; Gravemeijer, 1994)

Design research consists of three phases, namely developing a preliminary design, conducting pilot and teaching experiments, and carrying out a retrospective analysis (Gravemeijer, 2004; Bakker, 2004). Before describing these three phases, we need to define a Hypothetical Learning Trajectory (HLT). According to Bakker (2004), HLT is a design and research instrument that proves useful during all phases of design research. More detail, Simon (1999, in Bakker, 2004) defined HLT as follows:

The hypothetical learning trajectory is made up of three components, namely the learning goal, the learning activities, and the hypothetical learning process—conjecture of how students' thinking and understanding evolves in the context of the learning activities.

During the preliminary design, HLT guided the design of instructional materials that had to be developed or adapted. During pilot and teaching experiments, the HLT functioned as a guideline for the teacher and researcher what to focus on in teaching, interviewing, and observing. During the retrospective analysis, HLT functioned as guideline in determining what the researcher should focus on in the analysis (Bakker, 2004). In the following section, we discuss three phases of the design research according to Gravemeijer (2004), Bakker (2004), and Gravemeijer and Cobb (2006).

1. Preliminary Design

In this phase, the result was a formulation of what is called *a conjectured local instruction theory*, that is made of three components: learning goals for students; planned instructional activities and the tools that is used; and a conjectured learning process in which one anticipates how students' thinking and understanding could evolve when the instructional activities were used in the classroom.

2. Pilot and Teaching Experiments

a. Pilot Experiment

The pilot experiment acted as bridge between preliminary design and the teaching experiment. The aim of this pilot experiment was to determine whether the activities in the initial Hypothetical Learning Trajectory (HLT) were doable or not. The initial HLT was tried out and observed to be used as considerations to make adjustment of HLT in length measurement activity using running race route context as a starting point of the instructional activities.

b. Teaching Experiment

In this phase, instructional activities were tried, revised, and designed on a daily basis during the experiment. The actual regulation of the instructional activities in the classroom enabled the researcher to investigate whether the actual learning process corresponded with the one they had anticipated. The insights and experiences gained in this experiment formed the basis for the modification of the instructional activities and for new conjectures of students' thinking.

c. Retrospective Analysis

In this phase, all data during experiment were analyzed. The form of analysis of the data involved an iterative process, where the purpose of this retrospective analysis, in general, was to develop a local instruction theory. In this phase, the HLT was compared with students' actual learning. On the basis of such analysis, then we answered the research and sub-research questions and contribute to the local instruction theory.

B. Research Subjects and Timeline

Thirty-seven students (i.e., 5 students in pilot experiment and 32 students in teaching experiment) and a teacher of grade five in an Indonesian primary school in Palembang – Indonesia, SDN 179 Palembang, were involved in this research. The students were about 10 to 11 years old and they had learnt about the meaning of fractions in grade 3 and operation in fractions (addition and subtraction of fractions) in grade 4. SDN 179 Palembang had been involved in the Pendidikan Matematika Realistik (PMRI) or Indonesian Realistic Mathematics Education project since 2009. The organization of this research is summarized in the following timeline.

Table 4. The Timeline of the Research

	Date	Description
Preliminary Design		
Studying literature and designing initial HLT	21 September 2010 – 5 January 2011	
Discussion with teacher	24 - 25 January 2011	Communicating the designed HLT
Pilot Experiment		
Observation in grade 5 (class where students will be involved in the pilot study)	26 - 27 January 2011	Investigating social interaction among students. From the observation, we will choose 5 students to implement the initial HLT in small group.
Pre-test	28 January 2011	Investigating five students' pre-knowledge.
Trying out the sequence of activities	31 January – 4 February 2011	Trying out the initial HLT about partitioning, symbolizing fractions, relation among fractions, equivalence fractions, and multiplication fractions with natural numbers.
Post-test	7 February 2011	Investigating the presence of operational fractions concept (multiplication fractions with natural numbers) .
Teaching Experiment		
Pre-Test	22 February 2011	Investigating students' pre-knowledge.
Locating flags and	24 February 2011	Making construction of partitioning,

water posts on the running route activity		part of a whole.
Notating fractions in the empty fractions cards, putting the fraction cards on the string of yarn, describing the relations among fractions activities	1 February 2011	Symbolizing the result of partitioning and describing the relations among fractions such as equivalence of fractions.
Math congress I	3 March 2011	Sharing students' ideas and experiences in partitioning the track, symbolizing the result of partitioning, and the relations among fractions, fractions equivalency.
Determining who is running farther activity	8 March 2011	Comparing fractions within a certain length and the use fractions as multipliers.
- Math congress II - Minilesson (fractions as multipliers)	10 March 2011	- Sharing students' ideas and experiences in informally using fractions as multipliers and discussing several big ideas which may appear from the previous activity. - Encouraging decomposing and the use of partial products.
Post-test	15 March 2011	Investigating the presence of operational fractions concept (multiplication fractions with natural numbers).

C. Data Collection

The data collected in this research were interviews with the teacher and the students, classroom observations including field notes, and students' works. After we collected all data, we analyzed these data in the retrospective analysis. The data collections are described in the following table.

Table 5. The Description of Data Collections

	Data collection	**Description**
Part 1: Preliminary Design	• Classroom observation in Grade 5 *Video Recording*	• Investigating social interaction among students. • Finding socio norms and socio-mathematical norms. • Finding students' current knowledge about multiplication fractions with natural numbers.

	• Interview with grade 5 teacher *Audio recording*	• Finding students' current knowledge about multiplication fractions with natural numbers.
Part 2: Pilot Experiment	• Pre-test *Students' works*	• Finding students' current knowledge about multiplication fractions with natural numbers.
	• Trying out the sequence of activities (5 activities) with 5 students (small group). *Video and audio recordings, field notes, students' works*	• Testing initial HLT • Investigating students' strategies of solving problem in a sequence of activities which lead them to the idea of multiplication fractions with natural numbers.
	• Post test *Students' works*	• Investigating the presence of operational fractions concept (multiplication fractions with natural numbers) after students have experienced from the sequence of activities of multiplication fractions with natural numbers.
Part 3: Teaching Experiment	• Pre-test *Students' works*	• Finding students' current knowledge about multiplication fractions with natural numbers.
	• Trying out the sequence of activities (5 activities) with 5 students (small group). *Video and audio recordings, field notes, students' works*	• Testing HLT which has been revised. • Investigating students' strategies of solving problem in a sequence of activities which lead them to the idea of multiplication fractions with natural numbers.
	• Post test *Students' works*	• Investigating the presence of operational fractions concept (multiplication fractions with natural numbers) after students have experienced from the sequence of activities of multiplication fractions with natural numbers.
	• Interview with some students *Audio recording*	• Finding students' remark about the whole teaching and learning processes and special moment which occur in the learning activities.

D. Data Analysis

The data consisted of transcriptions of critical protocol segments from the students' taped instructional sessions, the student's written work, and detailed notes we made after each of the students' instructional sessions. The data also included transcriptions of critical protocol segments from students in small group (the first cycle) and some students in the whole group (the second cycle) and the teachers. Doorman (2005) mentioned that the result of a design research was not a design that successfully works but the underlying principles explaining how and why this design works. Hence, in the retrospective analysis the HLT compared with students' actual learning to investigate and to explain how students acquired their beginning understanding of multiplication fractions with natural numbers within length measurement activities.

We described students' learning processes of the way the students solve the problem and the reasoning of why students used a particular strategy either when their individually written note (math diary and student's individual work) or when they, in a small-group, gave arguments in the math congress. Additionally, we described the situations where the students needed assistance in understanding and solving a problem and the nature of the assistance that proved helpful. Furthermore, we compared the students' strategies, explanations, and questions about specific problems during the instructional activities. We asked opinion about the analysis to our supervisors. We discussed the analysis intensively and tried to improve it.

Finally, we made conclusions based on the retrospective analysis. These conclusions focused on answering the research questions. We also gave recommendations for the improvement of the next HLT, for mathematics educational practice in Indonesia and for further research.

E. Validity and Reliability

The validity concerns the quality of the data collection and the conclusions that are drawn based on the data. The data were collected throughout the learning activities that were designed to help students develop a beginning understanding of multiplication of fractions with natural numbers. Based on Bakker (2004), validity is divided into two definitions namely internal validity and external validity. Internal validity refers to the quality of the data collections and the soundness of reasoning that led to the conclusion. To improve the internal validity in this research, during the retrospective analysis we tested the conjectures that were generated in each activity and other data materials such as field notes, students' works, and interviews (source triangulation). Having these data, allowed us to conserve the triangulation so that we could control the quality of the conclusions. We also analyzed the succession of different lessons to test our conjectures of students' thinking and learning process. Besides, we also transcribed critical protocol segments of the video recordings to provide a rich and meaningful context.

External validity is mostly interpreted as the generalizability of the result (Bakker, 2004). It addresses the ability to generalize our study to other people and other situations. If lessons learned in one experiment are successfully applied in other experiments, this is a sign of successful generalization.

Reliability addresses as the consistency of the results obtained from the research. In Bakker (2004), reliability is also divided into two definitions namely internal and external reliability. Internal validity refers to the reliability within a research project (Bakker, 2004). In this design research, we improved our internal reliability by discussing the critical protocol segments in the design experiments with our supervisors and colleagues (peers examination).

External reliability usually denotes replicability, meaning that the conclusion of the study should depend on the subjects and condition, and not on the researcher (Bakker, 2004). Replicability is mostly interpreted as virtual replicability which the criterion of 'trackability' (Gravemeijer & Cobb, 2001; Maso & Smaling, 1998; Bakker, 2004). This means that the reader must be able to follow the track of the learning process in this research and to reconstruct their study. In order to do so, we provided two cameras which were called static and dynamic camera. Static camera was placed in a position where it could reach all the views when teacher and students interacted. Second, we used dynamic camera. This camera was handled by the research's assistance to take special moments when the learning process takes place. Besides, we also observe every moment in the classroom and make notes in the field notes. The result of the interviews with teachers and students, the teaching and learning activities, and the special moments were transcribed and analyzed.

CHAPTER 4
LOCAL INSTRUCTION THEORY

Based on the explanation in the section of Methodology, there are three phases in design research, namely: 1) preparing for the experiment, 2) experimenting in the classroom, and 3) conducting retrospective analysis. From a design perspective, the goal of the preliminary phase (phase one) of design research experiment is to formulate a local instruction theory that can be elaborated and refined while conducting the intended design experiment (Gravemeijer & Cobb, 2006). Local instruction theory consists of conjectures about a possible learning process, together with conjectures about possible means of supporting that learning process. We tried to anticipate how students' thinking might evolve when the planned but revisable instructional activities were used in the classroom. Learning goals for students are also the components of the conjectured local instruction theory (Gravemeijer, 2004). Therefore, in this chapter, we also elaborated the goals of this study.

A. The Goals

1. The Research Goal and the Research Questions

The goal of the research was to investigate the development of students' learning multiplication of fractions with natural numbers through different levels. In the sequence of activities in this study, we used contextual situation namely a running route. We also used tools and models for learning multiplicative thinking of fractions such as yarns, the string of yarn, and the number line which relate to the context.

Underneath, we raised a research question:

How does students' learning of multiplication fractions with natural numbers develop through different levels?

Moreover, we tried to focus the issues concerning different levels in learning multiplication of fractions with natural numbers. Therefore, we attempted to answer sub research questions, namely:

1. *How does the length measurement activity with the help of yarn provoke students in producing their own fractions?*
2. *How does the string of yarn lead the students to the idea of number line?*
3. *How does a number line lead the students to the idea of multiplication of natural numbers by fractions by generating equivalencies?*
4. *How does the involvement of certain length as the mediating quantity lead the students to the idea of multiplication of fractions by natural numbers?*
5. *How does student's own production about the idea of multiplication of fractions lead them on the way to rules for multiplying fractions by natural numbers?*

2. Learning Goals for Students

Through length measurement activity within contextual situations, the learning goals for the students are formulated as follows.

a. Students make a construction of partitioning, part of a whole.
b. Students symbolize the result of partitioning (notate fractions) and show it on the string of yarn.
c. Students will describe the relations among fractions such as equivalent fractions.
d. Students will construct multiplicative reasoning of fractions within equivalent fractions.
e. Students will compare fractions within a certain length.
f. Students informally use fractions as operator.

B. Conjectures about Possible (Mathematical) Learning Processes

Through this design research, we conducted a sequence of activities that develop a beginning understanding in multiplying fractions with natural numbers. In this design research, we emphasized the shift from mastering the algorithm to understanding the problem.

Partitioning a running route and symbolizing the result of partitioning were used as a starting point to assess the extended to which students understand the meaning of fraction itself where in Indonesian curriculum students had learnt about it in grade three. The idea of fractions as a part of a whole underlaid at the beginning of the activities. The concept of equivalent fractions led students' attention on multiplicative reasoning of fractions in which ratio and proportion played an important role in solving problems involving multiplicative relations. The context of '*determining who is running farther*' reconceptualized students' understanding of natural number multiplication to include fractions as multipliers.

Models also played an important role in this design research. In this sequence of activities, we used yarn, the string of yarn, and number line as models. The yarn was used in the first and in the second activities. The string of yarn was used as generalizing model in partitioning the running route. Then the number line was used as an abstraction of the string of the yarn which was in more formal level.

Yarn was used as tools to help the students in constructing partitioning of running route. With these tools, students could support their reasoning about the idea of part of a whole (the result of partitioning). The string of yarn could support students' reasoning about equivalent fractions which showed by fractions in the same position. The conjecture was that the students would aware if two fractions are equivalent, the ratio between the numerators is the same as the ratio between the denominators. At this moment, the development of ratio and proportion concepts is important for solving problems involving multiplicative relation. It was also conjectured that students would connect the idea of string of yarn to a number line which could be used as a generalization model.

In the transition from multiplication by natural number which the product is larger than the factors to multiplication by fractions which the product may either be higher or lower than the factors, a context of '*determining who is running farther*' was used. The conjecture was that students would include fractions as multipliers as they worked with fractions and natural numbers.

C. Conjectures about Possible Means of Supporting that Learning Process

Below, we describe the tools and their imagery in the learning activity shown in table 6.

Table 6. Tools and Imagery of the Learning Activity.

Tool	*Imagery*	*Activity*	*Potential Mathematical Discourse Topics*
The yarn	The running route	Locating flags and water posts on the running route.	• Measuring the length of the running route using yarn. • Informal knowledge of partitioning certain length into some equal parts.
The symbolizing of the portioned yarn	Part of a whole	Notating fractions on the empty fractions cards	Symbolizing fractions
The string of yarn and fractions cards	Generalizing tool	Describing the relation among fractions	The relation among fractions, i.e., equivalent fractions
The number line	Abstracting tool	Discussing the relation among fractions	The relation among fractions, i.e., equivalent fractions and multiplication of natural numbers by fractions.
The double number line	Abstracting tool	Determining who is running farther	Operation in fractions by attaching natural numbers which lead to the idea of multiplication of fractions by natural numbers.

In order to motivate the students to follow the sequence of activities, the contextual problems based on realistic mathematics educations were developed. The first contextual problem was about constructing partitioning of running route. In this activity students were asked to divide the running route into six and eight parts equally in order to locate the flags and the water posts in the running route. Teacher provided yarn. Hopefully students would aware that they needed that tool to help them in dividing the route as teacher challenged them if they could make the route in one straight line since the route is winding. In this case, the yarn was the model of the situation of the running route.

Another tool used in this sequence of activity was the string of yarn as a generalizing tool of the straight line. By using the string of yarn, it would support students to come up to the idea of number line and will see the idea of equivalent fractions. Equivalent fractions would support students' attention on multiplicative reasoning. Fractions involved between- and within- multiplicative reasoning. Within-multiplicative reasoning relation referred to the relation between the numerator and the denominator of fractions. Between-multiplicative relation referred to the relation between the numerators and the denominators of equivalent fractions.

After students had constructed their multiplicative reasoning, the instructional activities continued to the transition from multiplication by natural numbers to multiplication by fractions which supported them to reconceptualize the concept of multiplication by fractions in which the result might either be higher or lower than the factors. At this moment, it was conjectured that they would realize the use of fractions as multipliers.

D. Hypothetical Learning Trajectory (HLT)

Simon introduced the term, "hypothetical learning trajectory," which he describes as: "the consideration of the learning goal, the learning activities, and the thinking and learning in which the students might engage" to characterize the teacher's thinking (Simon, 1995). The mathematical teaching cycle might be described as conjecturing, enacting, and revising hypothetical learning trajectory.

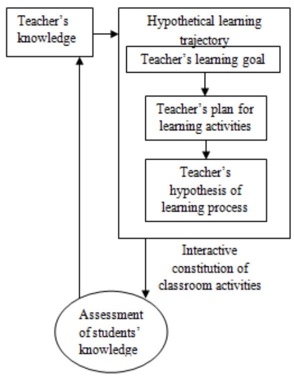


Figure 2. Hypothetical Learning Trajectory (Simon, 1995)

Based on the explanation above we will give detail of the learning activities and the visualization of the learning lines as follows.

1. The Learning Activities

In these activities, we tried to encourage the students to use concrete materials, draw pictures, or use other ways that were meaningful for him/her to solve the problems. The students were free to solve problems by using traditional algorithm for multiplication by fractions, in more specific multiplication fractions with natural numbers. However, we did not explicitly encourage or discourage the students from using this algorithm. We took this approach for several reasons: (a) to determine if the students possessed only knowledge of performing step-by-step procedures for the traditional algorithm or if this knowledge was connected to concepts in some way; (b) to avoid any possible interference we might impose by suggesting that the students used or did not use traditional algorithm; and (c) to determine if the students' knowledge of traditional algorithm influenced her/his solution processes in any way (Mack, 1995).

In each activity, students were asked to put down their own strategies in solving the problems in their so-called math diaries. Math diaries are little logs in which the students documented their learning processes and products. Their writing was *private* writing in a sense that they recorded their ways of working for themselves. However, their writings are also *public* writing because their recordings would be discussed in groups of students.

The discussion gathered in a so-called math congress in groups of four students which provided them with a good opportunity to present and discuss different ways of working. The goals of the math congress were twofold: students' abilities to communicate mathematically would be strengthened, but the social dimension of the learning would also be emphasized and serve as a counterweight to an overload of individualization (Selter, 1998). In the beginning of the instructional session, a guideline gave a brief explanation about how to conduct the math congress: 'try to solve the problems, put down your strategy in your diary in a way that can easily be understood by others, meet in small groups, present and discuss your productions and, if necessary, revise them.' After having conducted several math congresses, the students were asked to describe them in so-called writing conferences.

To support students' learning processes in developing a beginning understanding of multiplication fractions with natural numbers, we design a sequence of activities which consists of 6 activities. This sequence of activities is inspired from Context for Learning Mathematics (Fosnot & Hellman, 2007) and Mathematics in Context books (Holt et al.,

2003) with modifications. The hypothetical learning trajectory was elaborated based on the five activity levels (Streefland, 1991) which became the instructional activities as follows.

a. Producing Fractions

<u>Activity 1: “Locating Flags and Water Posts on the Running Route”</u>

Goal

Students make a construction of partitioning, part of a whole.

Materials

Student’s worksheet 1 (a set per group of students), the map of the running race (appendix A), yarn (about one meter per group), two colored markers, cartoons, scissors, notebook for ‘math diary’ (each student is asked to prepare a notebook), and a paper glue.

Description of the Activity

The context is about Ari and Bimo who train together to know their running ability for the running race. They run start from Palembang Indah Mall following the route to Palembang municipality office as shown in figure 3. This activity is done in group of four students. Each group make a poster of some of the things that they want to share in the discussion session. In the first session, the picture below is displayed. Each group of students also would get the map of the running route (appendix A, printed in A4 size).

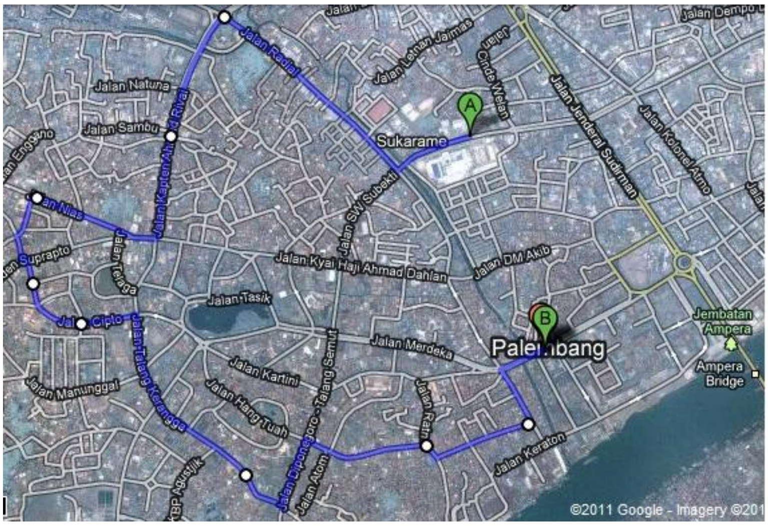


Figure 3. Running Route
Note: image is taken from *http://maps.google.com/*

At the same time, students were listening the following story:

> *To prepare running competition in the celebration of Indonesian Independence Day, Ari and Bimo practice their running skills. They plan to run from Palembang Indah Mall (point A) to Palembang municipality office (point B) following the running route. Eight flags and 6 water posts are stored on the track to know the position where Ari and Bimo will stop. Flags are placed on the running route with the same distance. The water posts also are placed on the running route with the same distance. The last flag and the last water post are stored at the finish line (in front of Palembang municipality office).*

The problem of the story was to *locate eight flags and six water posts on the running route which had to be equally spaced along the track.*

Teacher holds an initial discussion before the students work on this problem such as in order to locate the flags and the water posts, they need to divide the track into eight and six parts equally. Teacher reminds the students to put all their ideas in their own 'math diaries'. The aim of providing the students by yarn is to be used by the students as a tool to help them in making partitioning in order to locate the flags and the water posts. However, teacher should not interference the students to use the materials. Let them realize that they will need the materials in order to help them to solve the problem. As students struggle to partition or divide the track into eight and six parts equally, challenge students if they can make the track into one straight line. Confer with pairs of students as needed to support and challenge. Challenge strategies to become inquiries and to be generalized (Fosnot & Hellman, 2007).

Conjecture of Students' Thinking and Expectation

In order to locate the flags and the water posts, the students need to divide the track into six and eight parts equally. Here are a few of strategies we might see:

- If students do not use the materials, they may use estimation to partition the running route. For instance, without knowing the total length of the track, to find the position of 8 flags, first they may estimate the middle of the track.
- Students may use materials (e.g., the yarn) to know the length of the track then use it to make partition on the track. First they may map the yarn on the track. After that, they may straighten the yarn in order to get the track in one straight line. At this moment, teacher can pose question: *'How do you divide it (the yarn) into six and eight parts equally?'*. The way they partition the yarn may vary. For instance, to find the location of eight flags, first students may fold the yarn into two, fold it again into two, then the last fold it again into two in such a way that they get eight parts which the lengths in each part were equal. In this strategy students need to fold the yarn into two parts equally three times. At this moment, their ability in partitioning is required. After they partition the yarn, they may put mark on the folded parts, and then map them back into the running route in their drawing. The idea of partition by folding the yarn also can be used to find the position of six water posts.

Discussion

In the discussion session, first students present their works. They also observe their friends' works. The posters are sticked to the wall. At this moment, there will be a discussion about students' struggle in dividing the running route into six and eight. If there are students who use estimation (without using the tool), question that can be posed to those students is *'how do you convince yourself that the way you divide (partition) the track will get the right position?'*. If there are students who use yarn or paper strips in order to know the track in one straight line, the discussion will continue to the stretched the yarn which will bring them to the idea of number line (will be learned in activity 2). In the discussion, students can also explore the language of fractions of each partition that they made. For instance, do they realized when they divided one track into eight parts equally, each part represents one-eighth (an eighth)? Also when they divided one track into six parts equally, each part represents one-sixth (a sixth)? This question could provoke students to conclude that if a certain length was divided into some parts, each part represented one of those parts.

Activity 2: Notating Fractions in the Empty Fractions Cards, Putting the Fraction Cards on the String of Yarn, Describing the Relations Among Fractions

Goals

- Students symbolize the result of partitioning and show it on the string of yarn.
- Students will describe the relations among fractions such as equivalent fractions.

Materials

Student's worksheet 2, whiteboard, markers, empty fractions cards (each group gets one set of 14 empty cards which consists of 8 red cards and 6 blue cards), yarn that has been used to locate flags and water posts in the first meeting, and cellophane tape (one tape each group).

Description of the Activity

This activity will be done in group of four students. In the first session, the students are asked to symbolize/notate the result of partitioning that they have done in the first activity in the empty fractions cards. Because their posters are posted on the wall, so they still can see their work.

After students has sufficient amount of time to symbolize the result of partitioning into eight and six parts, they are asked to put their fractions cards on the yarn as they might work with it in the first activity in Day One. They may hang fractions card into yarn based on the order of the fractions and also their strategies in making partition of eighth and sixth which have been done in the first activity. Students are also asked to find the relation among fractions in their string of yarn. Teacher let the students to use their own words when describing the relations among fractions which appear when they work on putting fraction cards on the string yarn. Ask them to put their fraction cards which are hanging on the yarn below their poster (their work in activity 1).

After the activity of putting fractions cards on the string of yarn, students are asked to fill in student's worksheet 2. In the question 3, students (work in group) are asked to draw their stretched yarn and the fractions cards. Then in the next question they are asked to describe their findings from the drawing of yarn and the fractions cards. Teacher can provoke students by posing question: *'What do you find from your drawing in number 3? Note the position of the fractions'*.

Conjecture of Students' Thinking and Expectation

When notating fractions, probably there are some students who still use unit fractions (for instance, give a name $\frac{1}{8}$ for every part when partitioning the track into eight parts equally) instead of non unit fractions (for instance, $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, etc). Every part of the partitioning is symbolized by using unit fractions. For students who use unit fractions, teacher can guide them to the idea of non unit fractions. By the non unit fractions, the discussion is brought further to the idea of the number line of fractions and equivalent fractions which will be discussed in the discussion session. At this moment students may have a conflict about unit and non-unit fractions.

When the students are asked to describe their findings from the drawing of yarn and the fractions cards, it is expected that they will relate it with relations among fractions. It is also expected that they will find two fractions in the same position. Students may come to the idea of $\frac{3}{6} = \frac{4}{8}$. However, they may struggle in giving a reason why $\frac{3}{6}$ was equal to $\frac{4}{8}$. Because students have learnt about equivalent fractions in grade three and four based on Indonesian curriculum, KTSP 2006, probably there are students who will connect it to the idea of fractions equivalency. Students also may not see the relations among fractions if they do not make a proper partitioning.

Discussion

The discussion focus on students' drawing of the string of yarn and the fractions cards which hang on it. Furthermore, the discussion will continue to the idea of relations among fractions. Firstly, the teacher provoke students by posing question: *"from the fractions cards which is hanging on the yarn, what you can say about* $\frac{3}{8}$*? What is the relation between* $\frac{3}{8}$ *and* $\frac{1}{8}$*?".* The teacher let the students to explore the relation among fractions by themselves. It is conjectured that they would come up with the idea of multiplication of fractions as repeated addition which was in more formal could be said as

$\frac{3}{8}=3\times\frac{1}{8}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$. Besides, teacher can challenge the students to find fractions in the same position. They may struggle in giving a reason such as why $\frac{3}{6}$ was equal to $\frac{4}{8}$. The mathematical idea is if two fractions are equivalent, the ratio between the numerators is the same as the ratio between the denominators – to maintain equivalence, the ratio of the related number must be kept constant. The ideas of multiplication by fractions as repeated addition and the ratio must be kept constant will be discussed further in the Math Congress I.

b. Generating Equivalencies

<u>Activity 3: Math Congress I</u>

Goals

- Students will share their ideas and their experiences in partitioning the track, symbolize the result of partitioning, and describe the relations among fractions i.e. equivalent fractions.
- Students will construct multiplicative reasoning within equivalent fractions.

Description of the Activity

As a reflection of activity 1 and 2, at this moment teacher holds a class discussion. Teacher can discuss about the strategies of the students in solving problem in activity 1 and 2 which will provide a discussion of several big ideas such as:

- Fractions can be considered as measurement. It is obvious when students try to find the total length of the track, they do the measurement activity.
- Making fractions means dividing something into some parts which is equal.

- Making connection to a number line when students draw the stretched yarn which has been hung by fraction cards (Activity 2). Through this activity students do not only understand the unit fractions, they are also trained with non unit fractions.

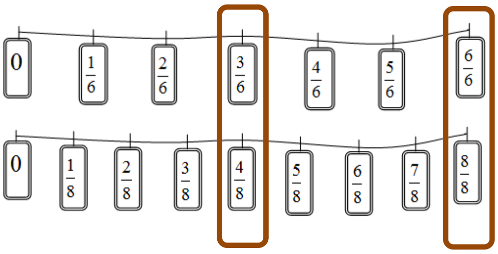


Figure 4. Fraction Cards Hang on the String of Yarn

From the fractions cards which were in the same position, i.e., $\frac{3}{6}=\frac{4}{8}$ and $\frac{6}{6}=\frac{8}{8}$, teacher can also talk about the equivalent fractions (figure 4). As stated in VanHille & Baroody (2002), teachers can also focus students' attention on multiplicative reasoning of fractions as they teach equivalent fractions. If two fractions are equivalent, the ratio between the numerators is the same as the ratio between the denominators. Besides, it also can be discussed about multiplication of fractions as repeated addition when students work on finding the relation between fractions, for instance the relation between $\frac{5}{8}$ and $\frac{1}{8}$ in the number line. Teacher can use the word 'jumps' to provoke students to the idea of repeated addition. Teacher can pose question: '*how many* $\frac{1}{8}$*-jumps start from zero point to* $\frac{5}{8}$ *?*'.

Teacher can draw the relation between $\frac{5}{8}$ and $\frac{1}{8}$ in the number line as follows (figure 5).

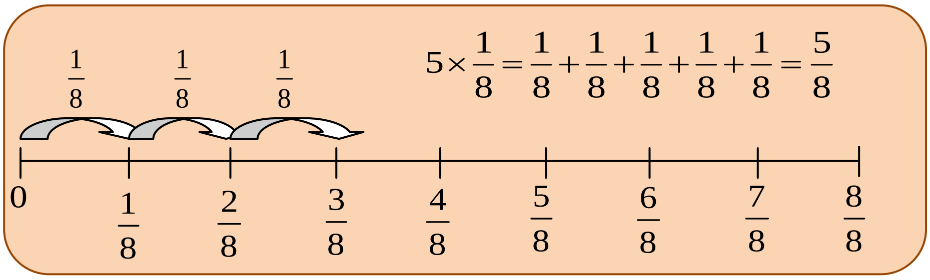


Figure 5. Relation Between $\frac{3}{8}$ and $\frac{1}{8}$ in the Number Line.

3. Operating Through a Mediating Quantity

<u>Activity 4: Determining Who is Running Farther</u>

Goals

- Students will compare fractions within a certain length.
- Students informally use fractions as multipliers.

Materials

Student's worksheet 4, whiteboard, and marker.

Description of the Activity

This activity will be done in group of four students. This problem still related to the activity 1 story. Continuing the story in the activity 1, to open this meeting, students are listening to the following story.

After all flags and water posts are in its position, Ari and Bimo start their training. They know the track length from Palembang Indah Mall to Palembang municipality office is 6 kilometers. After running for a while, Bimo decides to stop because he is exhausted. He stop at the fifth flag. Ari also decides to stop at the fourth water post.

The problem of this story is: '*How many kilometers have Bimo and Ari run*? *Explain your answer'*.

Teacher holds an initial discussion before the students work on this problem. Teacher reminds them to put all their ideas in their own 'math diaries'. At this moment, teacher must make sure that all students can understand the situation and what is the problem that needs to be solved. Teacher asks some students to recall back the story using their own language and give attention to the something which is important in the story. In this case the sentence: '*Bimo stops in the fifth flag. Ari stops in the fourth water post'* plays an important role of this problem and also the length of the running race route (6 kilometers). Remind the students that there are eight flags and six water posts in the running route. Challenge students to come to the idea of *five of eighth* and *four of sixth*? How they present it in the fraction symbols?

Conjecture of Students' Thinking and Expectations

Observe how students determine their answers, noting which students used more formal notations. Students can reason using the relations among fractions to solve this problem. Then students shall consider what represent the whole and how the parts are related to the total length of the running route. This strategy may lead the creation of a double number line (Holt et al., 2003). At this moment teacher can only pose questions which can provoke students' thinking when they have difficulties. Teacher can also introduce double number line model to the students. Teacher only gives a short clue then let the students develop and continue the next idea. For instance, teacher can draw double number line as follows (figure 6).

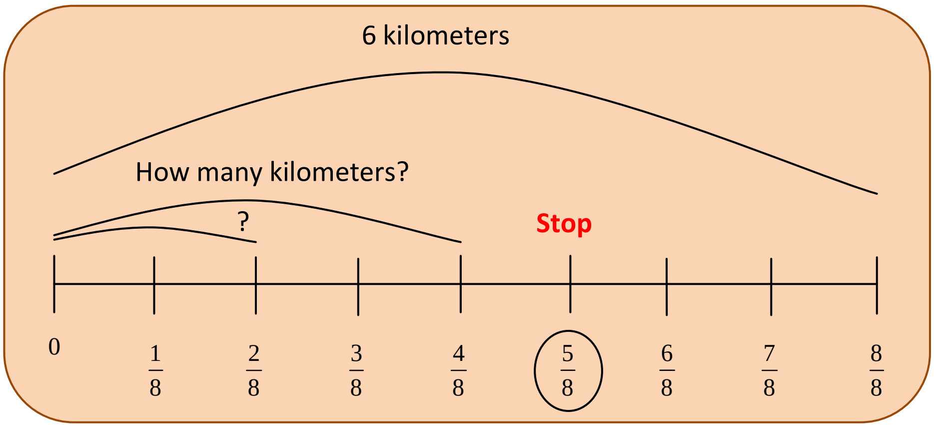


Figure 6. The Creation of Double Number Line

The students may solve the first problem about '*How many kilometers have Bimo and Ari run*?'. To solve this problem, they need to find *five-eighth* and *four- sixth of 6 kilometers.* Students may struggle to translate *four-sixth* and *five-eighth* of 6 kilometers into more formal notation. It was expected that the students would use fractions symbols of *five-eighth* ($\frac{5}{8}$) and *four-sixth* ($\frac{4}{6}$) as they have experienced it in notating/symbolizing fractions – in activity 2. In solving this problem, students might remark:

- To find $\frac{5}{8}$ of 6 kilometers. First, find $\frac{1}{8}$ of 6 kilometers which is $\frac{3}{4}$ kilometers, then times it by 5 (or using repeated addition with $\frac{3}{4}$ as an object which they want to add as many as five times). Similarly for $\frac{4}{6}$ of 6 kilometers. Student may use the double number line to show the repeated addition.
- To decompose $\frac{5}{8}$ into $\frac{4}{8}$ plus $\frac{1}{8}$ or in this case teacher can provoke students about the idea of fractions equivalency, so they can transform $\frac{4}{8}$ into $\frac{1}{2}$. In more formal way,

they may write $\frac{4}{8}+\frac{1}{8}=\frac{1}{2}+\frac{1}{8}$, then multiply it by 6, the length of the running route. They may come to the distributive property (*note: do not tell this term to the students*) as follows.

$$\left(\frac{1}{2}+\frac{1}{8}\right)\times 6=\left(\frac{1}{2}\times 6\right)+\left(\frac{1}{8}\times 6\right)=3+\frac{3}{4}=3\frac{3}{4}\text{kilometers.}$$

- To use proportional reasoning.

  If $\frac{1}{2}$ of 6 kilometers is 3 kilometers, then $\frac{1}{4}$ of 6 is $\frac{6}{4}$ kilometers or $\frac{3}{2}$ kilometers or $1\frac{1}{2}$ kilometers , and $\frac{1}{8}$ of 6 is $\frac{3}{4}$ kilometers.

After the students solve the first problem, they can determine 'who is running farther' by only seeing how far Bimo and Ari had run.

4. Doing One's Own Productions

<u>Activity 5: Math Congress II</u>

In this meeting a math congress is held. Discussion focused on how decomposing numbers and using partial product could be helpful. The mini lesson in the next meeting will allow students to revisit the strategies which have been discussed in the congress.

Goals

- Students will share their ideas and their experiences in informally using fractions as operator.
- Students will discuss several big ideas which will appear in the discussion.

Materials

Student's worksheet 5, student's works of activity 4, whiteboard, and markers.

Description of the Activity

Teacher holds class discussion. The variety of strategies from students answer from the previous activity is likely to provide a discussion – the big ideas (Fosnot & Hellman, 2007):

- Fraction can be decomposed into unit fractions or ‘friendly’ fractions.
- Fractions can be considered as multipliers on other numbers. For instance, when students need to find ‘*How many kilometers have Bimo and Ari run?’*. They need to find $\frac{5}{8}$ of 6 kilometers and $\frac{4}{6}$ of 6 kilometers.
- The whole matters. In comparing $\frac{5}{8}$ and $\frac{4}{6}$, students not always compare the numerator or the denominator.
- The distributive property holds for multiplication over addition for fractions. Partial products ($\frac{1}{2}\times 6$ and $\frac{1}{8}\times 6$) could be used to determine the product of $\frac{5}{8}\times 6$.
- Using proportional reasoning, students may determine that if $\frac{1}{2}$ of 6 kilometers is 3 kilometers, then $\frac{1}{4}$ of 6 is $1\frac{1}{2}$ kilometers, and $\frac{1}{8}$ of 6 is $\frac{3}{4}$ kilometers.

Conjecture of Students’ Thinking and Expectation

This running route is similar to a number line. It is expected that the number line model helps students to develop a conceptual understanding of fractions as numbers. It is also expected that the students will give reasoning with fractions by using the relations among the given fractions. They also will multiply the given fractions by total distance of the running route (6 kilometers) to compare relative distance of each fractional part of the running route. This final strategy hopefully will help students to develop a conceptual understanding of fractions as multipliers.

If there are no anwers from students which lead to double number line model where fractions and the length of the running track located on the number line, the teacher can introduce the use of the double number line as one strategy in solving the problem. Teacher does not introduce the whole process in solving the problem using double number line model. For instance, teacher only provokes the students by asking their strategies in partitioning the running track into eight parts equally.

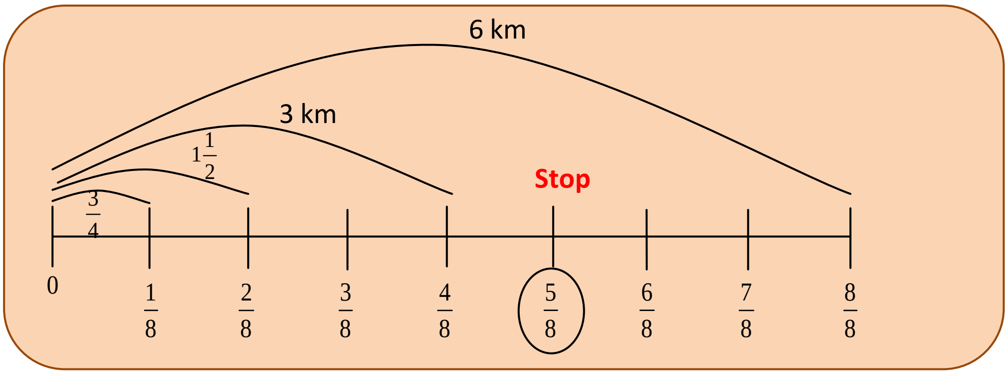


Figure 7. The Double Number Line Model

From the double number line model, the conjectures of student's thinking answer in determining $\frac{5}{8}$ of 6 kilometers are:

- If the students relate the double number line with the idea of jumping on the number line of fractions, they would add $\frac{3}{4}$ five times. It is because there are five jumps from zero point to $\frac{5}{8}$. Example of the calculation is as follows.

$$\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=\frac{15}{4}$$

From repeated addition of fractions, students can relate this into multiplication of natural number with fractions which can be written as $5\times\frac{3}{4}$. At this moment teacher can provoke the students to change fractions into mixed fraction so that $\frac{15}{4}$ will produce mixed fraction $3\frac{3}{4}$.

- Students only add 3 kilometers with $\frac{3}{4}$ kilometers. It is because to reach $\frac{5}{8}$, it has been taken half way which is 3 kilometers then they just need to add one jump more which the length was $\frac{3}{4}$ kilometers.

5. On the Way to Arithmetic Rules for Fractions

Activity 6: Mini lesson: Fractions as Operator

The string of numbers in the mini lesson is crafted to encourage decomposing and the use of partial products. It also can allow students to revisit the strategies which have been discussed in the congress.

Goals

- Students are able to make their own word problem from the string of number in this minilesson.
- Students are given opportunity to use their strategies in solving the problems from the previous activity.

Materials

Student's worksheet 6, whiteboard, and marker.

Description of the Activity

Teacher opens the learning process by reviewing the matters that had been discussed in the Math Congress 2. Teacher gives some times to the students to solve all problems in this students' worksheet 6.

After the students finish all the problems in this worksheet, the teacher discusses about the big ideas and strategies in solving the problem. As students offered strategies, modeled one of the strings on a double number line, and invited them to a discussion. From the representation using the double number line, it is expected that it might help students realize that the partial product ($\frac{1}{2}\times 34$, $\frac{1}{4}\times 34$ and $\frac{1}{8}\times 34$) can be used to determine the product of the whole ($\frac{5}{8}\times 34$). At the end of this activity, teacher reminds the students to write their experience working on the activities today in their math diaries.

<u>The String of Numbers</u>

$\frac{1}{2}\times 34$ $\quad$ $\frac{1}{8}\times 34$

$\frac{1}{4}\times 34$ $\quad$ $\frac{5}{8}\times 34$

Table 7. Sequence of Activities of Multiplication Fractions with Natural Numbers.

Learning Goals	Mathematical Ideas	Activities	Students' Struggles	Students' Strategies
a. Students make a construction of partitioning, part of a whole.	- Fractions represent a relation: making partition means dividing certain length into some parts.	Locating flags and water posts	- To divide track into six and eight parts equally first they need to know the total length of the track. - When they do not use materials that are provided, they struggle in making estimation in partitioning the track into six and eight parts.	- Using estimation (without using the materials). - Using yarn to know the total length of the track. Yarn was folded into two first (half of yarn and paper folding) as the first step to find six and eight parts equally.
b. Students notate the result of partitioning and show it on the string of yarn. c. Students will describe the relations among fractions i.e. equivalent fractions.	- Recall back the concept of unit and non unit fractions. - Fractions that are in the same position are equal fractions. - Multiplication by fractions as a repeated addition can be introduced within equivalent fractions.	- Notating the result of partitioning in the empty fractions cards. - Show the result in a string of yarn which fraction cards hang on it. - Describing the relations among fractions which students can see from yarn with fraction cards hang on it.	- Understanding non unit fractions. - Conflict in deciding the position of fractions on the yarn. - Giving reason what is the relation between certain fractions such as $\frac{3}{8}$ and $\frac{1}{8}$. - Giving a reason such as why $\frac{3}{6}$ is equal to $\frac{4}{8}$. - They may not see the relations among fractions if they do not make a proper partitioning.	- Give fractions' symbol by ordering the fraction. - Symbolize each part of partitioning with unit fractions. - Students will see relations among fractions such as $\frac{3}{6}=\frac{4}{8}$ and may relate it to the idea of fraction equivalency and also the relation between $\frac{3}{8}$ and $\frac{1}{8}$ which will lead them for the beginning of the using multiplication of fractions as they work on fractions in repeated addition such as $\frac{3}{8}=3\times\frac{1}{8}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$.
d. Students will share their ideas and their experiences in partitioning the track, symbolizing the result of partitioning, and the relations among fractions.	- Fractions can be considered as measurement. - Making fractions means dividing something into some parts which is equal. - Making connection to a number line. - To maintain equivalence, the ratio of the related	Math Congress I	- When determining the ratio between numerator and denominator of two fractions which are equivalent.	- In constructing multiplicative reasoning of fractions within fractions equivalency, students may only look at the numerator first (both two fractions which are equal) then the denominator.

e. Students will construct multiplicative reasoning of fractions within equivalence of fractions.	number must be kept constant.			
f. Students will compare fractions within a certain length. g. Students informally use fractions as multipliers.	- Fractions as multipliers. - Decomposing fractions and the use of partial product. - Multiply fractions with natural numbers.	- Determining who is running farther. - Math Congress II - Minilesson: Fractions as Multipliers.	- Students may struggle in translating four of sixth of the whole distance of the track and five of eighth of the whole distance of the track. - In calculating $\frac{4}{6}$ of 6 kilometers and $\frac{5}{8}$ of 6 kilometers.	- To find $\frac{5}{8}$ of 6 kilometers. First, find $\frac{1}{8}$ of 6 then times it by 5. Student may use the double number line to present the repeated addition. - To split $\frac{5}{8}$ into $\frac{4}{8}$ plus $\frac{1}{8}$ which will use the idea of equivalent fractions so that they can transform $\frac{4}{8}$ into $\frac{1}{2}$. In more formal way, they may write $\frac{4}{8}+\frac{1}{8}=\frac{1}{2}+\frac{1}{8}$, then multiply it by 6. They may come to the distributive property (*note: do not tell this term to the students*) as follows. $\left(\frac{1}{2}+\frac{1}{8}\right)\times 6=\left(\frac{1}{2}\times 6\right)+\left(\frac{1}{8}\times 6\right)=3+\frac{3}{4}=3\frac{3}{4}$ kilometers. - To use proportional reasoning. If $\frac{1}{2}$ of 6 kilometers is 3 kilometers, then $\frac{1}{4}$ of 6 is $1\frac{1}{2}$ kilometers, and $\frac{1}{8}$ of 6 is $\frac{3}{4}$ kilometers.

2. VISUALIZATION OF LEARNING LINES OF MULTIPLICATION FRACTIONS WITH NATURAL NUMBERS

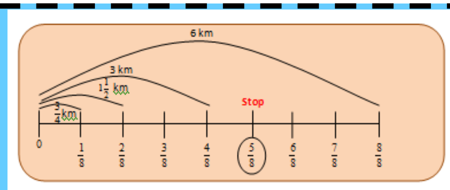
Learning Line

$\frac{5}{8}\times 6$ $\frac{4}{6}\times 6$

$\frac{5}{8}=5\times\frac{1}{8}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$

- Fractions as operator
- Multiplication fractions as repeated addition

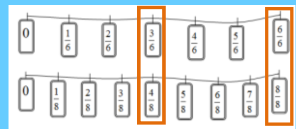
Multiply fractions with natural Numbers



Fractions as operator

Comparing fractions within a certain length

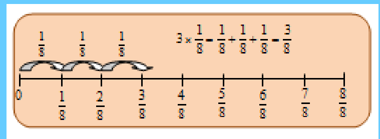
Determining who is running farther



Constructing multiplicative reasoning of fractions within equivalence of fractions

To maintain equivalence, the ratio of the related number must be kept constant

Relation among fractions



Recall back the concept of unit and not unit fractions

Notating fractions

Fractions cards

- Fractions represent a relation
- Fractions can be considered as measurement

Making constructions of partitioning

Locating Flags & Water posts

Note : Mathematical ideas (rectangles), Learning goals (rounded rectangles), Activities (ovals)

CHAPTER 5
RETROSPECTIVE ANALYSIS

In this chapter, data collection from both the pilot and the teaching experiments were described. The hypothetical learning trajectory that had been designed in chapter 4 was used as a guideline in the retrospective analysis to investigate the development of students' learning multiplication of fractions with natural numbers through different levels.

A. Pilot Experiment

This pilot experiment was conducted in a group of five students, excluded from teaching experiment class. The list of the students was given in table 1. This pilot experiment aimed to analyze and evaluate the initial hypothetical learning trajectory. In addition, input was also obtained from students' struggles in working out the sequence of activities in the HLT. The result of this pilot experiment would improve the initial HLT.

Table 8. List of Students in the Pilot Experiment.

No.	Name	Class
1.	Andini	V-C
2.	Fachrie Azzumar Dwiyan	V-A
3.	Ghalda Putri Balqis	V-A
4.	Oriang	V-C
5.	Zulfa Kamila	V-A

The designed Hypothetical Learning Trajectory (HLT) that was tested to these five students consisted of pre-assessment and six activities. The result of this pilot experiment will be explained based on the sequence of instructional activities as follows.

1. Pre-assessment

Pre-assessment were designed to assess students' initial abilities about fractions and to know students' strategies in solving the problems. Students were given a written test consisted of five problems. The problems in this pre-assessment were about partitioning a certain length, ordering fractions, describing the relation among fractions, and doing operation with fractions.

Three out of five students used estimation strategy to determine the point where they could make partition. The last two students used their ruler to measure the length of the shape then divided it into several parts which were asked in the problems. This showed that the students realized the need of the length of the shape to partition it into several parts.

In answering the problem of ordering fractions, most students did not have any difficulties. They could easily fill in fractions which were in its order.

About the problem of finding the relation among fractions, the students looked to the numerator and denominator of the fractions. For instance, when they were asked to see the relation between $\frac{1}{5}$ and $\frac{4}{5}$, most of the students answered these fractions had same denominators but different numerators. Our expectation from these findings is that the students will come up with the idea of multiplication of fractions, e.g. *four times of* $\frac{1}{5}$ *will get* $\frac{4}{5}$ which can be seen from the numerators.

The last problem about operation with fractions, all students thought that this problem was about division of natural numbers. They did not realize the use of fraction which appeared in this problem. They directly divided 9 meters of ribbon by three to find the length of brown ribbon. They knew the brown ribbon was a third of ribbon that had been bought by Risa.

From the explanation above, we can conclude that the students have learnt about fractions in more formal way. It can be seen when they were doing the problem about ordering fractions, they could easily answer it because fractions in formal notation appeared in the problem. They also directly talked about numerator and denominator when they were asked to find the relation among fractions.

After having discussion with supervisor and colleagues, it seemed necessary to change this pre-assessment because this pre-assessment was seen as giving the students a picture of the activities that they would do in the next meetings. The revised pre-test would consist of general questions about fractions. The materials were the meaning of fractions (fractions as part of a whole), equivalent fractions, order fractions, and operation in fractions. These materials were the materials needed to support the understanding of the concept of multiplying fractions, especially multiplying fractions with natural numbers and vice versa.

2. Activity 1: Locating Flags and Water Posts on the Running Route

The goal of this activity was to see students' ability to construct a partitioning of a certain length. The problem was to locate eight flags and six water posts which had to be equally spaced along the running track. This problem gave opportunity to the students to produce their own fractions through length measurement activity.

In general, students' strategies in locating flags and water posts were match with our formulated conjectures, namely students used ruler or yarn or both as a helpful tool to know the length of the running track, students folded the yarn into eight and six parts equally, and students remapped again the yarn after they gave marks on it. At the first step, students tried to measure the length of the running track by mapping the yarn following the track as shown in figure 8.

Figure 8. From Left to Right: Andini, Oriang, Zulfa, Fachrie, and Ghalda Tried to Map the Yarn Following the Running Track.

These were students' strategies in partitioning the running track after knowing the length of the track using yarn:

1. Using ruler to find the total length in centimeters then tried to find out how many centimeters each parts by using the concept of division of natural numbers as shown in figure 9.

Figure 9. Fachrie Tried to Measure the Total Length of the Route

2. Folding the yarn into some parts. Then put marks on the folded part as shown in figure 10.

Figure 10. Zulfa Folded Her Yarn then Gave Marks on It

After the students finished dividing the track into eight to locate the flags, they remapped the yarn back to the running route then gave flags which represented flags. With the same steps the students found out the location of water posts.

After trying out the activity of locating flags and water posts, there were some inputs for the improvement of worksheet 1. Observations suggested the following be revised in worksheet 1.

1. Running Track Map

It was observed that the students had difficulty in following the path of the running route (the left side with a very small turn). Therefore, we tried to change the running route from Palembang municipality office to Palembang municipality office again become from Palembang Indah Mall (PIM) to Palembang municipality office. This opening activity is very essential because if the students did not consider the need of precision in the measurement activity, they could not continue to the next activity, where in the next activity they would symbolize the result of partitioning. This first activity was repeated in the next meeting using new running track.

2. The Selection of the Tools

It was observed that each student got different result in measuring the length of the track using wool yarn. This was probably because of wool yarn's flexibility. Therefore, we will replace wool yarn with a regular yarn to reduce students' error in measuring.

In this activity, we also provided Japanese ribbon with the aim to help students if they had difficulties in folding the wool yarn. However, this actually confused the students because too many tools were provided. Therefore, we did not provide Japanese ribbon again in the next meeting. We only selected the set of tools which were yarn, scissors, and two colored markers for this first activity.

3. The Language and the Legibility

Because of the changes of the running route, the story in Worksheet 1 was also changed as follows.

To prepare running competition in celebrating Indonesian's Independence Day, Ari and Bimo practice their running skills. They plan to run from Palembang Indah Mall (point A) to Palembang municipality office (point B) following the given running track. Eight flags and six water posts are stored on the track to know the position where Ari and

Bimo will stop. Flags and water posts are respectively placed on the running route at the same distance. The last flag and the last water post are stored at the finish line (in front of Palembang municipality office).

Besides, to avoid ambiguity in question number 3, we revised it as follows.

From the story above, locate:

a. The flags which have to be equally spaced along the track.
b. The water posts which also have to be equally spaced along the track.

3. Repetition of Activity 1: Locating Flags and Water Posts on the Running Route

This activity was a repetition of the first activity where the running track had been revised in order to minimize students' error in measuring the length of the running track using yarn. Based on the plan, we only provided yarn, scissors, and two colored markers. Without providing Japanese ribbon, students could still fold the yarn into eight and six parts equally then marked the folded parts using colored markers. Using the new running track, most students could find the location of water post and the flag which are in the same position (even not as precise as expected) which had been described in our conjectures. This finding would guide the students to the idea of equivalent fractions which would be discussed further in the next meeting after the students symbolized the result of partitioning. One of students’ work in locating flags and water posts which indicated flag and water post in the same position can be seen in the following figure (figure 11).

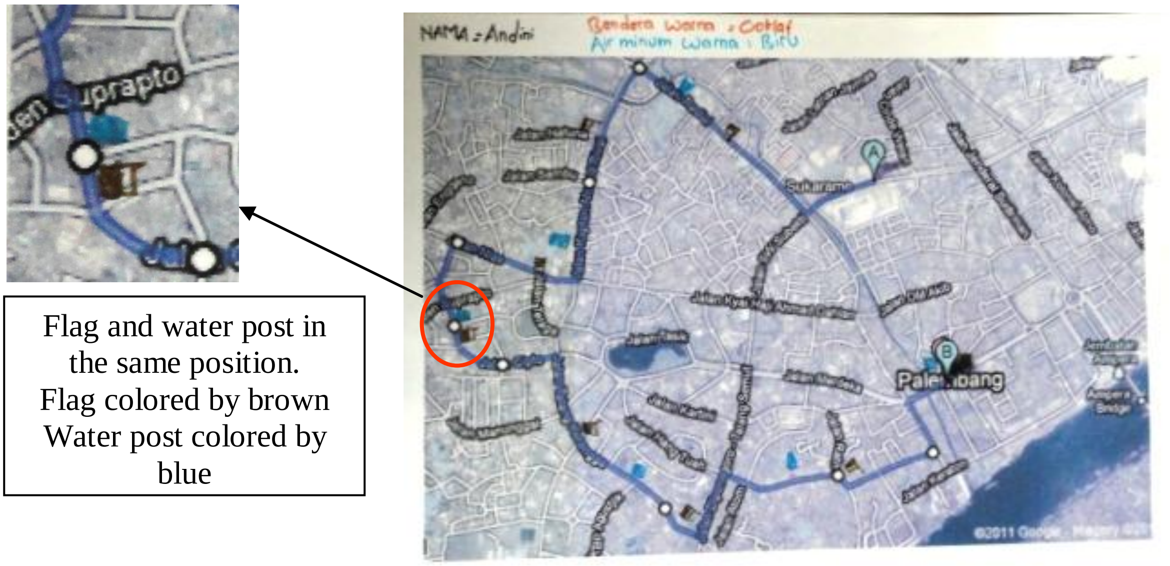


Figure 11. One of Student's Worksheet in Locating Flags and Water Posts in the Running Route.

The part of finding flag and water post in the same position is one of important part based on five activity levels which precede the learning operation in fractions (Streefland, 1991). This finding match to the conjecture that was formulated based on the literature as discussed in chapter 2, namely the generating equivalencies as students were asked to determine fractions in the same position. The idea of ratio and generating equivalencies would be discussed further in activity 2.

4. Activity 2: Notating Fractions in the Empty Cards, Putting the Cards on the String of Yarn, Describing the Relations Among Fractions

The goals of this activity were that the students could notate the result of partitioning in the empty cards, put it (the cards that had been written by fractions) on the string of yarn and describe the relations among fractions such as equivalent fractions which can be seen from fractions in the same position. It was observed that the students did not have any difficulties when putting fractions cards on the yarn since they had marked the position of flags and water posts in the first activity as shown in figure 12.

Figure 12. Fachrie Tried to Put Fractions Cards on the Marks.

In notating fractions in the fractions cards, one out of five students still used unit fractions (e.g. $\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6}$) as shown in figure 13, while the others had already come up with the idea of non unit fractions (e.g. $\frac{1}{6},\frac{2}{6},\frac{3}{6},\frac{4}{6},\frac{5}{6},\frac{6}{6}$) as shown in figure 14.

Figure 13. Ghalda Using Unit Fractions

Figure 14. Andini Using Non Unit Fractions

This findings match with our conjecture, namely in notating fractions, there was a student who still used unit fractions (for instance, gave a name $\frac{1}{8}$ for every part when partitioning the track into eight parts equally) instead of non unit fractions (for instance, $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, etc). In this activity, we let Ghalda realized herself that she would need non unit fractions to compare two fractions (problem in worksheet 4).

Next activity, students put their fractions cards on their yarn that had been used in the first activity as shown in figure 15.

Figure 15. Andini with Her Fractions Cards Hanging on the Yarn

In general, students had no difficulties in working out this second activity. They could find that there were two fractions in the same position. They defined it by using mathematics symbol '='.

However, it seemed necessary to have additional questions to bring the students into a discussion about multiplication of natural numbers by fractions as repeated addition. We tried to revise question number 5 into several questions as follows.

5. *Look at again your answer in number 3. How many* $\frac{1}{8}$*- jumps from starting point (zero point) to* $\frac{5}{8}$*? Explain your answer.*

It is expected that this question will provoke students who still using unit fractions ($\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}$) in notating fractions to realize the importance of non unit fractions to find the relation among fractions.

Next question, we tried to remind the students about the concept of multiplication of natural numbers as repeated addition (question number 6).

6. *What do you know about multiplication of* 2×3*? Give your reasoning.*

From the concept of multiplication of natural numbers, we hope that the students will come up with the idea of multiplication natural numbers by fractions when they are asked to find the relation between $\frac{1}{8}$ and $\frac{5}{8}$ (question number 7). By giving this question, it is expected that the students will able to write multiplication natural number with fractions in mathematical notation as they realized the relation between repeated addition of natural numbers and repeated addition of fractions. The question is as follow.

7. Explain what are the relation between $\frac{1}{8}$*-jumps with the point of* $\frac{5}{8}$ *!*

5. Activity 3: Math Congress 1

In this activity, teacher's creativity is needed to explore students' findings from the previous activities which can be brought into the discussion. In this first math congress, students were given opportunities to share their experiences in solving the problems in the first and the second activities. Discussion with the students yielded the following facts.

1. Notation of fractions.

Students notated their fractions by using unit fractions (e.g., $\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6}$ and $\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8}$) and non unit fractions (e.g., $\frac{1}{6},\frac{2}{6},\frac{3}{6},\frac{4}{6},\frac{5}{6},\frac{6}{6}$ and $\frac{1}{8},\frac{2}{8},\frac{3}{8},\frac{4}{8},\frac{5}{8},\frac{6}{8},\frac{7}{8},\frac{8}{8}$).

2. The idea of "number line of fractions".

Students discussed how to draw a string of yarn and the fractions cards. It was surprising that Andini came up with the idea of 'number line'. In addition, Zulfa called it 'a number line of fractions'. The following is a fragment from video recording about our discussion with the students.

Researcher : *Can you recognize what are they?* (pointing out students answer in question number 4

Students : *These are fractions.*

Researcher : *How about this?* (pointing the representation of yarn)

Andini : *It's a line. Because I drew a line to show this* (pointing her yarn). *I drew a rectangle and put arrow on both sides.*

Zulfa : *Me too. I also drew a rectangle.*

Researcher : *How about you Ghalda and Oriang?*

Ghalda : *Me? Hmm..I don't know. I made a very tiny rectangle.*

Oriang : *Ya, I also drew rectangle.*

Researcher : *Hm..most of you drew rectangle as a representation of yarn. Why? Zulfa, Fachrie, Ghalda, or Oriang, could you explain about it?*

(…)

Researcher : *Ghalda, why did you make a very tiny rectangle?*

Ghalda : *Because I think this (*pointing yarn*) has wide this big (*showing the width of yarn by using her thumb and forefinger where between thumb and forefinger there is a small gap). *So, I made a very tiny rectangle.*

Researcher : *Hm..I think it's reasonable. How do you think Zulfa?*

Zulfa : (She smiled). *Mm..I think Ghalda is right.Mine is too big* (she tried to erase her drawing because she thought her drawing was false)

Researcher : *No, no..I do not mean your drawing is incorrect. Hmm..here* (pointing the arrows in Zulfa's rectangle), w*hy do you make these arrows?*

Zulfa : *Mm..*(smile and shook her head as sign it was difficult to explain).

Researcher : *How about the others? What do you think?*

Fachrie : *Mm..it is because there are still some numbers after this point* (pointing $\frac{8}{8}$) *and also after zero point, there are negative numbers.*

Researcher : *Oh ya? Have you learned about negative numbers?*

Fachrie : *Mmm..ya, in the first semester.*

Researcher : *Hmm.. Andini, what do you think? why did you draw a line?*

Andini : *Hm..I don't know.* (She seemed still unsure with her answer, because most of her friends made rectangle as a representation of yarn)

Researcher : *Okay then, can we combine Ghalda's, Andini's, and Zulfa's thoughts about the shape representation of yarn, Fachrie's thought about negative and positive numbers which he get it from Zulfa's drawing of arrows? What do you think, what are we talking about now?*

Zulfa : *Hmm..about fractions but no negative number of fractions.*

Researcher : *Yup, it's true. Can you combine it by the representation shape of yarn?*

Zulfa : *Maybe it is a kind of number line. Because I think Andini's drawing is correct. She made a line as a representation of yarn.*

Researcher : *Hmm..How do you know this is a number line?*

Andini : *Yes! I think Zulfa right! It's a number line* (she sketched her number line which was consisted negative numbers, zero, and positive numbers)

Researcher : *Hmm..and now, do we have negative numbers in our fractions?*

Zulfa : *No, we start from zero. If this is a number line and wehave fractions, and… Hmm..a number line of fractions!*

Researcher : *Hm..I heard something from Zulfa, could you please repeat it again?*

Zulfa : *Because it's a number line and in this number line we see fractions. So, I called it a number line of fractions!*

From the conversation above, we can see students' process to come up with a more formal idea of '*number line of fractions'*. Students' representation of yarn can be seen in the following figures (figure 16 and figure 17).

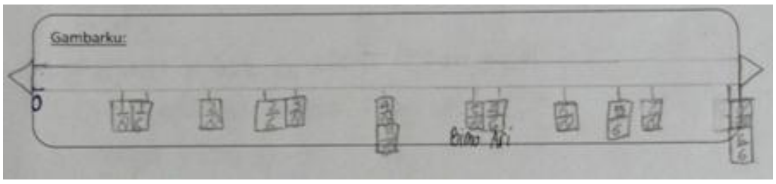


Figure 16. Zulfa’s Representation of Yarn and the Fractions Cards

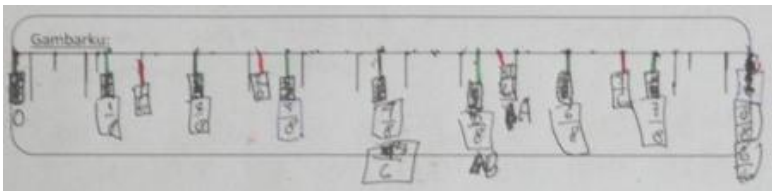


Figure 17. Andini’s Representation of Yarn and the Fractions Cards

3. Equivalent fractions.

Students found that fractions in the same position represents equivalent fractions such as $\frac{4}{8}$ and $\frac{3}{6}$ also $\frac{8}{8}$ and $\frac{6}{6}$ as shown in figure 18.

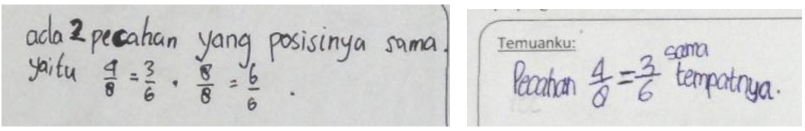


Figure 18. Fachrie’s and Oriang’s Answer about Fractions in the Same Positions.

One out of five students came up with the idea of simplifying fractions. He divided both numerator and denominator using same number. From his calculation, he realized the reason why $\frac{4}{8}$ equal to $\frac{3}{6}$. It was because those two fractions had same value which was $\frac{1}{2}$ as shown in figure 19.

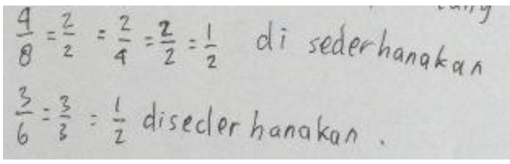


Figure 19. Fachrie’s Tried to Simplify Fractions.

From figure 19, we could see Fachrie's calculation in simplifying fractions. However, there looked strange. He used symbol '=' besides $\frac{4}{8}$ then wrote $\frac{2}{2}$. To know his thinking, some questions were asked to him as follows.

Researcher	:	*Fachrie, what do you mean by this?* (pointing $\frac{4}{8}=\frac{2}{2}$)
Fachrie	:	*Mm..* $\frac{4}{8}$ *both divided by 2.*
Researcher	:	*What do you mean by 'both'?*
Fachrie	:	*Number at the top and also number at the bottom both divided by 2. The result is* $\frac{2}{4}$*, then I divide it again by two because it still can be simplified. The result is* $\frac{1}{2}$*. This is the final answer.*
Researcher	:	*And how about this* (pointing '$=\frac{2}{2}$' besides $\frac{4}{8}$)*, what does it mean?*
Fachrie	:	*Hmm.. It is a division, I divide both sides, therefore I write* $\frac{2}{2}$*, 2 for number at the top and 2 for number at the bottom.*
Researcher	:	*And how about this symbol?* (pointing symbol '=' besides $\frac{4}{8}$)
Fachrie	:	*Ya, it is a division.*
Researcher	:	*Can you show me how you make a division sign?*
Fachrie	:	(He made a scratch. From his scratch, it showed a sign which was similar with equal sign '=')
Researcher	:	*Do you make division symbol like that?*
Fachrie	:	(Nodding his head). *Ya*.

From the conversation above, we know that Fachrie had problem in differentiating how to write the division symbols and equal symbols. Therefore, we tried to correct his writing of division symbols which should be ":". It is important since if he kept writing division symbols same with equal symbols, it can give another interpretation and may cause miscalculation.

4. Introducing the concept of multiplication as repeated addition by using familiar word '*jumps*' as shown in figure 20.

At this moment, we tried to provoke the students by giving the word 'jumps' as something that led to the multiplication. Then we let the students realized that if they started to jump from zero point to $\frac{3}{8}$, and each jump was $\frac{1}{8}$, so, there were three times jumps of $\frac{1}{8}$. From the picture above, it shows Fachrie's transition from the word 'jumps' to the mathematical notation '+'.

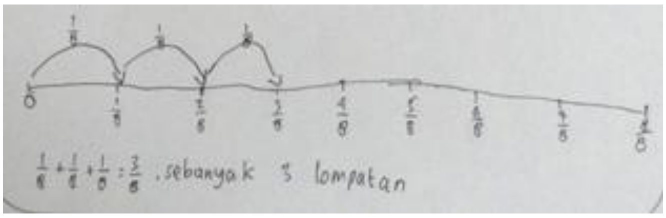


Figure 20. Fachrie's Idea of 'Jumps'

From figure 20, it was discussed further about addition of fractions with the same denominator. We tried to remind the students the way to solve addition of fractions. Most of the students had no difficulties in adding fractions with the same denominator. From this repeated addition we then tried to provoke the students about the concept of multiplication as repeated addition.

6. Activity 4: Determining Who is Running Farther

The goals of this activity were that the students could compare fractions within a certain length and informally use fractions as operator. Because of the changes of the running track, the story in this activity was also changed. We designed the running track with the same distance, 6 kilometer. From the observation, we found that at the end of the story there was a question which asked '*who is running farther, Ari or Bimo?*', and then the students could easily answer the question by marking the position of Ari and Bimo in their answer in worksheet 2. In fact, first we hope that the students would include the distance from Palembang Indah Mall (PIM) to Palembang municipality office with the given fractions. Therefore, we will eliminate the last sentence in the story.

It was observed that the students did not have any difficulties in translating the position of Bimo who stopped on the fifth flag into fractions which means $\frac{5}{8}$.

Researcher	:	*Do you know where Bimo stops?*
All students	:	*The fifth flag.*
Researcher	:	*So, what is the point of Bimo?*
All students	:	$\frac{5}{8}$.
Researcher	:	*Why do you say it is* $\frac{5}{8}$?
Oriang	:	*Because there are 8 flags and Bimo stops at the fifth flag, so this is the point where Bimo stops.* (pointing out the point where $\frac{5}{8}$ placed in his worksheet).

All students agree with Oriang's explanation.

It was observed that the students had difficulties in translating the question about *how many kilometers have Bimo and Ari run?*. They could not relate the question into the concept of fractions multiplication. For instance, they knew the position of Bimo was $\frac{5}{8}$ but they did not know how to find $\frac{5}{8}$ of 6 kilometers. Therefore, we tried to provoke them by introducing the use of double number line. We only gave a short clue about the use of double number line where they could put the distance (6 kilometers) above the number line and fractions below the number line. We asked students whether they could find a half of the distance from PIM to Palembang municipality office in the number line.

We tried to provoke the students to remember their strategies in partitioning the route into eight which had been done in the first activity. Two out of five students realized that they needed to divide 6 kilometers by 2 which gave a result 3 kilometers. The conversation then continued as follows.

Researcher	:	*Now, we know the length of a half of this* (pointing the number line). *So, what do you think? We want to know how many kilomenters from this* (pointing zero point) *to this* (pointing $\frac{5}{8}$).

Noone gave any reaction. We realize that this was the hardest part to include fractions in counting operation.

Reseacher : *Hm..do you remember how to divide the running route into 8 parts equally?*

Fachrie : *Ya, first, I folded it into two, then fold it again into two, and fold it again into two, then I get 8 parts equally.*

All the students agree with Fachrie's explanation. Then it was silent for a while.

Researcher : *And now, what do you think? If we know a half of this length, then how could we know the length of* $\frac{1}{8}$?

Ghalda : *Aha, You divide it again into two! Then divide it again into two!*

Researcher : *Why?*

Ghalda : *Because we follow like what we did in the first activity.*

Researcher : *But, remember, you want to find* $\frac{5}{8}$. *How do you find it?*

Andini : *I add* $\frac{1}{8}$ *five times.*

From the conversation above, it is observed that the students came up with the idea of proportional reasoning which is match with our conjectures. Moreover, Andini came up with the idea of multiplication as repeated addition as she said '*I add it five times'* as shown in figure 21.

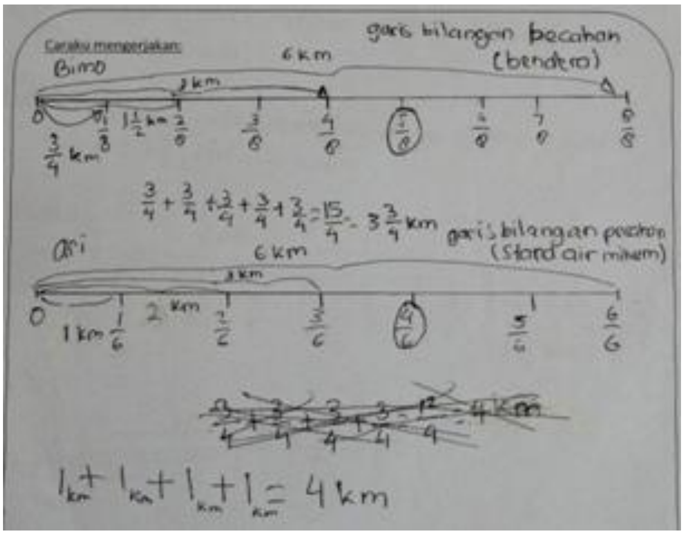


Figure 21. Andini's Work Using the Double Number Line Strategy

7. Activity 5: Math Congress 2

In this math congress, together with the students, we discussed about the use of double number line as one of strategies that could be used to solve problem in activity 4. In this math congress, the number of flags and the distance of running route were changed. There was an interesting answer from Fachrie as shown in figure 22. The following is a segment described the discussion with Fachrie.

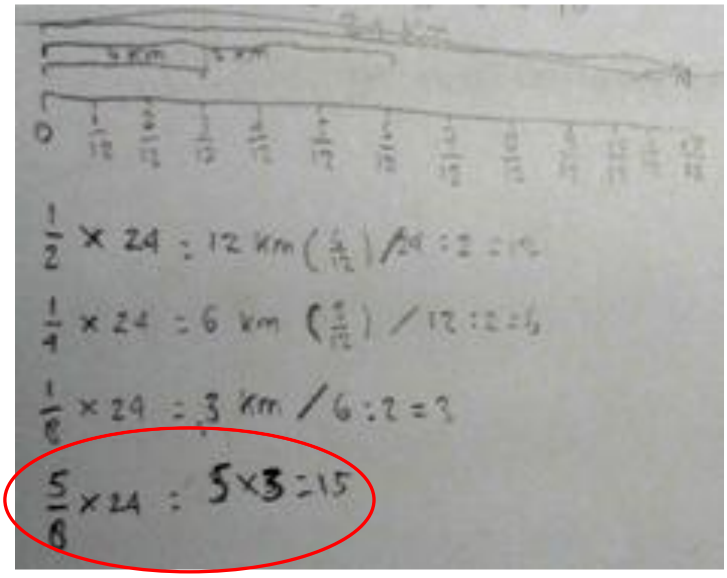


Figure 22. Fachrie's idea of proportional reasoning

Researcher : *Hm..Fachrie, could you explain what this means?* (pointing fachrie's note in his worksheet 5)

Fachrie : *You say that we change the distance of the running track is 24 kilometers, right? Then we also change the number of flags which are now become 12. We want to know the distance from start (zero point) to* $\frac{5}{8}$.

Researcher : *Then what does* $\frac{1}{2}\times 24$ *mean?*

Fachrie : *First, I want to know a half of 24 kilometers. It is in this point* (pointing $\frac{6}{12}$). *I know the distance by dividing 24 by two which gets 12 kilometers. I do the same thing. I want to know a quarter of 24 kilometers which the point is* $\frac{3}{12}$. *Then I divide 12 by two again. I get 6. And also to find* $\frac{1}{8}\times 24$, *I divide 6 by 2 and gets 3.*

Researcher : *Hm..then I was wondering how you came up with* 5×3 *when you wanted to find* $\frac{5}{8}\times 24$*?*

(silent for a while)

Fachrie : *Hm..it is because* $\frac{1}{8}\times 24$ *is 3 kilometers, then we want to know* $\frac{5}{8}\times 24$*, so, we only need to multiply 3 kilometers by 5. Because this is 5* (pointing 5 in the fraction $\frac{5}{8}$)

From the conversation above, it is observed that the student constructed his mathematical language of multiplication concept as he said *a half of 24 kilometers* which in his note, he wrote $\frac{1}{2}\times 24$. Looking at this strategy, we can conclude that Fachrie used the idea of proportional reasoning to find $\frac{5}{8}\times 24$, even he did not know the name of it. The idea of proportional reasoning appears in our conjectures in hypothetical learning trajectory.

8. Activity 6: Mini Lesson: Fractions as Operator

The string of numbers in the mini lesson allowed students to revisit the strategies which have been discussed in the congress. Problems given in this worksheet were already abstract, more in mathematical notation. Students were asked to make their own story related to the problem. The following was one of students' works in making story problem which still related to running competition but she changed the name of the players using her name and her friend's name (figure 23). This moment was in line with our conjectures, namely students still try to relate the problem with the running problem.

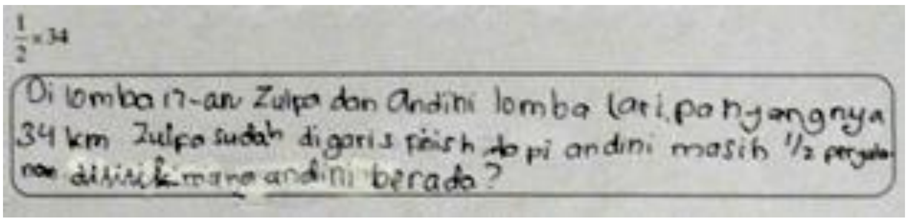


Figure 23. Ghalda's Story Problem.

The translation of Ghalda's story is as follows.

In Indonesian independence day's competition, Zulfa and Andini join running competition. The length is 34 kilometers. Zulfa has stopped in the finish line but Andini still in a half way. Where is the point of Andini?

From Ghalda's story, several question were asked such as 'what does she mean with *the length is 34 kilometers?*', 'what is *a half way*?', and 'what do you mean by asking *the point of Andini*? Is it point or the distance?. There was surprising moment when we had discussion with Ghalda which is explained as follows.

Researcher	:	*Ghalda, can you tell me your story of problem number 1?*
Ghalda	:	*In indonesian independence day's competition, Zulfa and Andini join running competition. The length is 34 kilometers. Zulfa has stopped in the finish line but Andini still in a half way. Where is the point of Andini?*
Researcher	:	*Hmm..I heard from your story about the length. What did you mean by length in this story?*
Ghalda	:	*Hmm...ya, it was 34 kilometers.*
Researcher	:	*Ya, what did you mean with 34 kilometers?*
Ghalda	:	*It was....hmm..for example, the distance from Palembang Indah Mall to Palembang municipality office os 34 kilometers.*
Researcher	:	*Okay, what do you think? Was it a length or a distance?*
Ghalda	:	*A length..hm..no, it was a distance from PIM to Palembang municipality Office.*
Reseacher	:	*Okay, then, what did you mean by Andini still in a half way?*
Ghalda	:	*Hmm..Zulfa had arrived the finish line while Andini still in a half way which is 17 kilometers.*
Researcher	:	*Wow, how do you get 17 kilometers?*
Ghalda	:	*I divided 34 by 2.*
Researcher	:	*Why?*
Ghalda	:	*I do not know.*
Researcher	:	*Hm...first you said Andini still in a half way, then you calculte the distance of Andini by dividing 34 by 2. What do you mean?*
Ghalda	:	*Hmm...ya, because Andini in a half way and it was in the middle of the way, half is the same as* $\frac{1}{2}$*,because this is 2 (*pointing 2 in the numerator*), so I divided 34 by 2.*

From the conversation above, it was observed that Ghalda come up with the idea of dividing natural numbers by denominator of fractions. This idea could also be seen from her work when she solved the first problem as shown in figure 24.

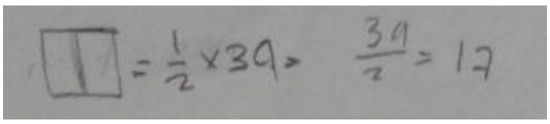


Figure 24. Ghalda's Answer of Problem Number 1.

To know Ghalda's thinking about the first problem, an interview was held. The following is a segment decribed the discussion with Ghalda.

Researcher	:	*Ghalda, could you explain your answer of question* $\frac{1}{2}\times 34$ *?*
Ghalda	:	*A half of 34 is 17 kilometers.*
Researcher	:	*And, what is the function of this drawing? (pointing Ghalda's drawing beside her answer* $\frac{1}{2}\times 34$*)*
Ghalda	:	*Ya, because it was a half, then I divided it into two. A half of 34 is 17.*
Researcher	:	*Hm..Ok, then how about* $\frac{34}{2}$ *?*
Ghalda	:	*Ya, because when I divided 34 by 2, the answer is also the same. So, I put 2 as the numerator. But, hey, can you see? It always works when I solved the next problem!* (pointing her answer of problem number 2,3, and 4 as shown in figure 25).

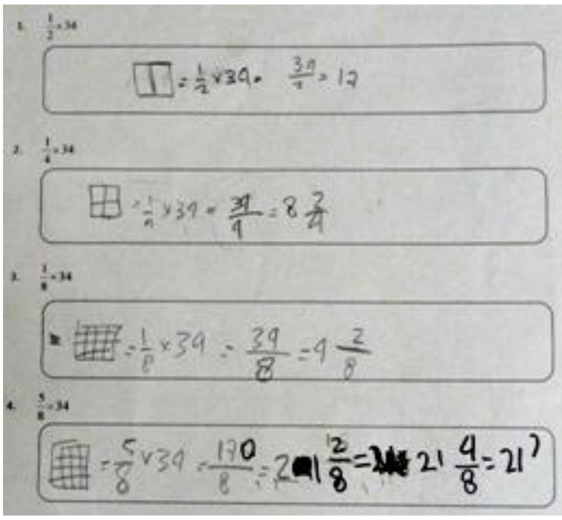


Figure 25. Ghalda's Answer about Multiplication of Fractions in Worksheet 6.

From the conversation and Ghalda's answer above, it is observed that Ghalda tried to construct her thinking about multiplication of fractions without using the context of running race route again. She tried to generalize her rule for all problems and in fact, her answer was true. This surprising moment was in line with the literature that had been described in chapter 2. The last activity levels which precede the learning operation in fractions (Streefland, 1991), namely on the way to rules for the operations with fractions. Within this string of numbers in this Minilesson, the student reflected on the rules for the multiplication by fractions operations. The transition to more formal fractions was preceded by stimulating students to contribute their own informal ways of working.

9. General Conclusion of the Pilot Experiment Activities

The tryout of initial HLT showed that the prospective subjects (i.e. grade 5 students) had already perceived the idea of partition certain length into some parts which then will be connected with fraction as part of a whole, the idea of unit and non unit fractions, The use of non unit fractions to compare fractions, and the use of fractions as operator when natural number involves in the multiplication fractions. However, to develop students' understanding about these concepts, there are some parts of activities which need to be revised for the improvement of initial HLT.

There are many findings obtained from the observation in this pilot experiment that had been explained in the previous section. From these findings, together with the teacher and colleagues, the designed HLT as well as students' worksheet and teacher guide are improved and adjusted to the fact that obtained in the field. The revision of initial HLT is named as HLT II. These instruments will be used in the next cycle of this research, namely the teaching experiment phase.

B. Teaching Experiment

The analysis of the teaching experiment phase was elaborated by five activity levels which preceded the learning operation in fractions suggested by Streefland (1991), namely producing fractions, generating equivalencies, operating through mediating quantity, doing one's own productions and on the way to rules for the operations with fractions. This aimed at investigating the progress of students' learning on multiplication of fractions with natural numbers through length measurement activity which afterwards could be generalized for instructional design.

Besides, pre-test and post-test were given to the students in the beginning and in the last meeting of this sequence of activities. The purpose of pre-test was to check students' pre-existing knowledge about fractions and the purpose of post-test was to measure students' understanding after they followed the learning activities.

1. Pre-assessment of Students' Knowledge

As described in sub chapter A, namely pilot experiment, we tried out the revised pre-test to thirty-two students who involved in this teaching experiment phase. Based on Indonesian curriculum, students in grade 5 had already learnt about fractions since in grade 3. They had learnt the meaning of fractions, equivalent fractions, ordering fractions, and operation in fractions (i.e., addition and subtraction of fractions). Those concepts had a role in this hypothetical learning trajectory. We tried to observe whether or not the students were able to follow the sequence of activities. There were four problems given in this pre-test to check students' understanding of fractions.

The aim of the first problem was to check students' ability to partition or to divide an object into several parts equally. In this problem, students were asked to divide the picture of pempek lenjer into four and three parts equally. They should also describe how they made the drawing. Most of students had no problem in partitioning pempek lenjer into four parts. About 88% of the students could partition pempek lenjer into three parts. However, there were different strategies to describe their drawings, namely using estimation and using ruler. The following (figure 26) were some students' answers in partitioning using estimation strategy.

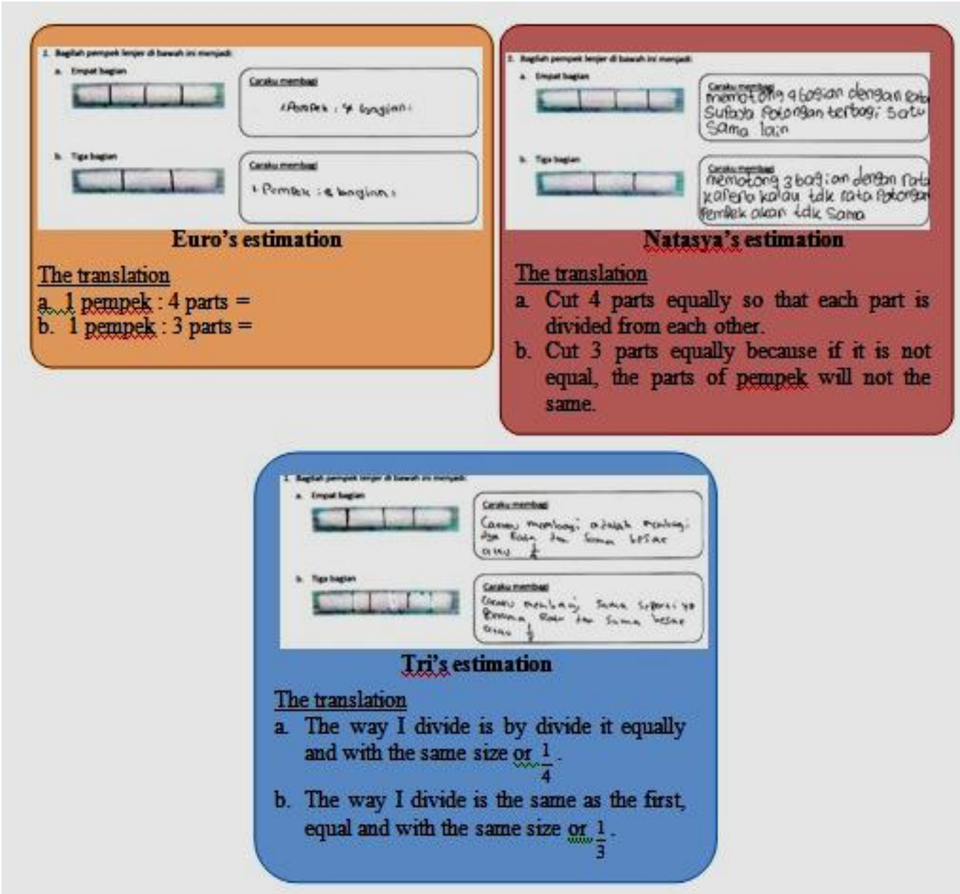


Figure 26. Some Examples of Students' Answers in Partitioning Pempek Lenjer into Four and Three Parts Equally.

To know their thinking in solving the problems, we tried to interview them. The transcription is as follows.

Researcher	:	*Euro, would you like to explain your way in dividing pempek into four parts?*
Euro	:	*I just estimated it. First divided it into two then divided the result into two again.*
Researcher	:	*How about you Natasya and Tri?*
Natasya	:	*Ya, I also did the same thing.*
Tri	:	*Ya, me too.*
Researcher	:	*Mmm..How about dividing pempek into three parts? Do you also estimate it?*
Natasya, Tri, and Euro	:	*Ya, we did it.*
Researcher	:	*How do you convince yourself that the result of your partitioning was correct?*
Euro	:	*Mm..I think mine was correct, because each part more or less has the same length.*
Researcher	:	*Hmm..I heard the words 'more or less'. What do you mean by more or less?*
Euro	:	*Mm..ya, more or less, because I don't know the length of each part exactly.*
Researcher	:	*Hmm..And what do you think? Do you know how to find the length of each part exactly? Does anybody know how to know the length of each part?*

Natasya tried to use her ruler and measured each part in her drawing.

Reseacher	:	*Hmm..Natasya, what are you doing?*
Natasya	:	*Mmm..I try to measure the length of each part by using my ruler.*
Researcher	:	*And how about the result?*
Natasya	:	*Mm..The length is not precisely the same. This part* (pointing the first part in the second drawing) *is too big than other parts.*
Researcher	:	*Mmm..Then what do you think?*
Natasya	:	*Ya, each part should have the same length. My drawing is incorrect.*
Researcher	:	*How about the others? What do you think, do you have any comment?*
Euro	:	*Mmm..I think without using ruler, as long as our estimation close to the exact length, then it's okay. The use of ruler was only to check whether our estimation was correct or not.*
Researcher	:	*Mm..I think your reason is acceptable Euro. And now, what do you mean by* $\frac{1}{4}$ *and* $\frac{1}{3}$ *here in your explanation, Tri?*
Tri	:	*Mmm..that's the result of each part. This part is* $\frac{1}{4}$ *and this part also* $\frac{1}{4}$*, and this part also* $\frac{1}{4}$.(pointing each part in the first drawing).
Researcher	:	*How do you come up with the idea of* $\frac{1}{4}$ *?*
Tri	:	*Because we have learnt about fractions and I think each part represent fraction.*
Researcher	:	*Wow, good job!*

From conversation above, we know that the students come up with the idea of the need of ruler to check their estimation when they doubted on the result of partitioning. They also come up with the idea of fraction notation $\frac{1}{4}$ and $\frac{1}{3}$.

Apart from the above strategy, there were also some students who used ruler to measure. One of students' answers is shown in figure 27.

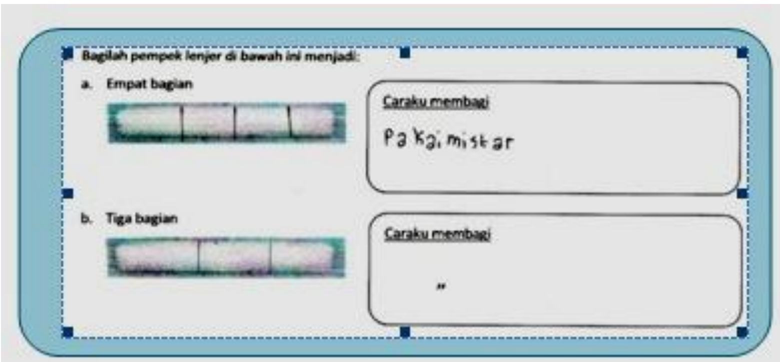


Figure 27. Dinda's Strategy in Partitioning Pempek into Four and Three Parts Equally by Using Ruler.

The rest of the students could not give description of their strategy. In spite of their various strategies in partitioning, we can conclude that students had ability in partitioning things into several parts. This ability would be required when the students worked on the first activity in sequence of activities later on.

The second problem was about finding equivalent fractions. Most of the students were able to find equivalent fractions. In the following is one of students' answers (figure 28).

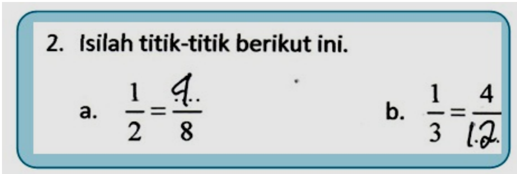


Figure 28. Natasya's Equivalent Fractions.

The idea about equivalent fractions would be used as the students worked on the next activity level, namely generating equivalencies. They would describe the relation among fractions from their own fractions that had been produced from the first activity level, namely producing fraction.

The third problem in the pre-test was about ordering fractions. The problem was related to our sequence of activities, namely putting fractions card on the string of yarn after students notating the result of partitioning. Therefore, from this problem, we could see students' ability in ordering fractions which would be used in the activity which related to ordering fraction.

Most students had no difficulties in answering this problem. However, there was one student who could not correctly answer the problem. It was because of student's error in reading the question. It was asked to order fractions from least to greatest. But this student arranged the fractions from greatest to least. Student's answer is shown in the following figure (figure 29).

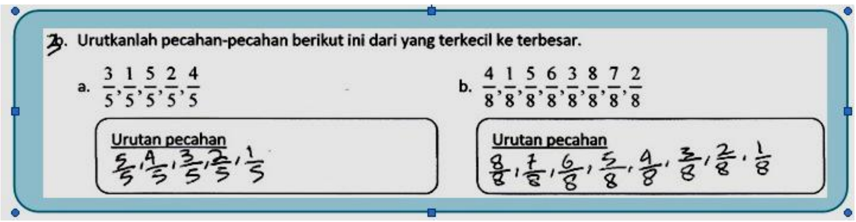


Figure 29. Reza's Answer about Ordering Fractions from the Greatest to the Least.

The last problem was about operation in fractions. By providing this question, we wanted to know whether or not the students could translate the problem which involved fractions in counting operation. Only one student successful solved this problem (figure 30).

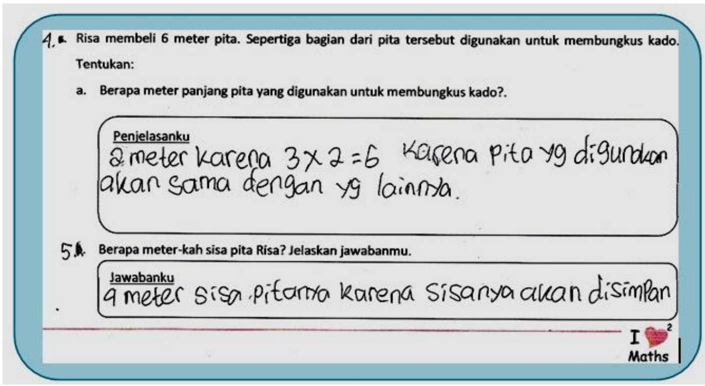


Figure 30. Natasya's Answer about Operation which Involved Fractions.

The translation of Natasya's answers is as follows.

a. 2 meters because $3 \times 2 = 6$ because the used ribbon will be the same with the others.

b. 4 meter the rest of the ribbon because the rest will be saved.

From Natasya's answer, we can see that she tried to answer the problem by working on the opposite direction. The reason that we called it by opposite direction, because she determined the answer was 2 meters, then she explained by checking it out again whether when she multiplied 2 meters three times she would get 6 meters. She translated the word 'a third' in the problem into division with natural number 'divided by 3'. In the following is a fragment of Natasya's thought about 'a third'.

Researcher	:	*Hm, Natasya, how do you find that the ribbon which used for wrapping the presents was 2 meters?*
Natasya	:	*Mm..because 6 divided by 3 is 2.*
Researcher	:	*What do you mean by 6 divided by 3?*
Natasya	:	*In the question, Risa has 6 meters ribbon and a third of it was used to wrap the present.*
Researcher	:	*Where did you get '3' while there's no '3' in the question?*
Natasya	:	*This one* (pointing the word of 'a third').
Researcher	:	*A third?*
Natasya	:	*Ya, a third, means divided by 3.*

From the conversation above, we can see that Natasya came up with the idea of division in natural numbers. She did not realize that this question was about multiplication fraction with natural number. However she could answer this question by a different thought.

We gave the pre-test to the students in orders to see students' prior knowledge about fractions. From the result of students' pre-test, we can conclude that students' knowledge about fractions was sufficient to follow the sequence of activities in our designed HLT.

In the next part, we will describe and explain the analysis of the result of our designed experiment. We will focus on five activity levels which precede the learning operation in fractions and also the topics which show interesting findings and appropriate with the expectations. We will also describe suprising moments that happen in the discussion with the students and students' challenges and struggles in doing the activities. Besides, we also will discuss some activities that need to be revised in order to achieve the goals behind the activities.

2. Producing Fractions

As mentioned in the first tenets of Realistic Mathematics Education, contextual problems figured as applications and as starting points from which the intended mathematics could come out. For that reason, the running race route context was chosen as the context in which the students could produce fractions by their selves within length measurement activity.

Measurement activity in this phase was seen when the students tried to measure the total length of the running route before dividing it into eight and six parts equally by using yarn (activity 1). The result of partitioning the running route would be notated into fractional symbol in the next activity (activity 2). In general, the expectations from these activities were students could make a construction of partitioning and they could produce their own fractions so that they could understand the meaning of fraction as part of a whole.

a. Locating Flags and Water Posts on the Running Route and Its Contribution in Supporting Students' Acquisition of the Concept of *Partitioning* a Certain Length

In the first activities, students worked in group and were asked to locate eight flags and six water posts in the running route. In this activity, students' skills in partitioning certain length were required. Each group was facilitated by yarn as a tool which could help them in knowing the length of the route. The students mapped the yarn following the running route as shown in figure 31 below.

Figure 31. Students Work Together to Map the Yarn on the Running Route.

From the picture above, we can see that students cooperated together to measure the total length of the running route. After they knew the length of the track, they cut the rest of the yarn which was not part of the length of the running route. However, when the teacher asked the length of the running route, there were some groups who misinterpreting the length of the running route from Palembang Indah Mall (point A) to Palembang municipality office (point B). They answered it was 10 centimeters because they measured the shortest distance by using ruler as shown in figure 32 below.

Figure 32. Students Measured the Shortest Distance from Point A to B.

A conflict occurred at that time. When this group said the length of the running route was 10 centimeters and another group justified their answer, there was a student who argued that they were wrong. They must follow the blue line in the map. Teacher performed her role at this time. She asked the students about the correct route and asked one of the students to follow the route which was shown by the blue line. This surprising moment showed that there was missing information in the story about the running route which made the story became ambiguous. Therefore, we needed to add some words in the story as follows.

They plan to run from Palembang Indah Mall (point A) to Palembang municipality office (point B) following the running route (the blue line).

After the students knowing the length of the route, they were asked to partition yarn into eight first (to know the position of the flags). The crucial guidance from the teacher is shown in the following fragment.

Teacher	:	*Now you know the length of the route is as long as the yarn that has been map to the running route. Look at the question 3.a, we want to know the location of eight flags. From your yarn, what will you do to get eight parts equally?*
Students	:	*We divide the yarn.*
Teacher	:	*How do you divide it?*

There was a student who tried to measure the length of the yarn by using ruler, but then he had difficulty in dividing it by eight because the total length was 57 centimeters. Suddenly, there was a student who found out that they needed to fold the yarn in order to divide the yarn into eight parts equally. Then the conversation continued as follows.

Nanda	:	*Aha, we need to fold the yarn!*
Teacher	:	*Nanda, why do you fold it?*
Nanda	:	*Because it is easier than divide 57 centimeters by 8. We only need to fold the yarn into two, fold it again, and fold again then we will get 8 parts equally.*
Teacher	:	*Good job Nanda!*

Nanda's idea of folding the yarn inspired other students. They did the same way to partition the yarn into eight as shown in figure 33 below.

Figure 33. A Student Tried to Fold the Yarn.

After the students folded the yarn, they gave signs to each folded parts by using marker then remapped the yarn back to the running route to give the signs of the flags' and water posts' location (figure 34 and 35).

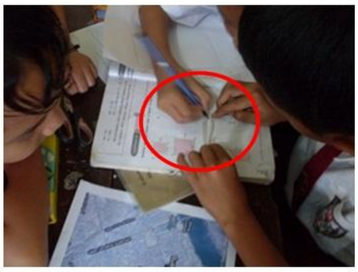


Figure 34. Students gave signs to the folded parts.

Figure 35. Students remapped the yarn back to give marks of flags' and water posts' location.

Most of the students could partition the yarn into eight parts equally. However, when they were asked to partition the yarn into six parts in order to locate six water posts, some of them had difficulties. Teacher provoked the students by giving a clue: '*if I fold the yarn into two, then it can produce two equal parts. Now, what should I do to get six equal parts?*'. From this provoking question, some students came up with the idea of folding the first folding part into three. They did trial and error strategy until they got six parts equally.

From locating flags and water posts on the running route activity, it can be concluded that the students could made a construction of partitioning by the help of yarn and the idea of folding the yarn. This finding was in line with our conjecture namely students used materials (i.e., the yarn) to know the length of the track then used it to partition the track. However, there was no students used estimation strategy to partition the route. The result of partitioning the running route will be notated in fractional symbol in the next activity, namely notating fractions in the empty fractions cards.

b. Notating Fractions in the Empty Fractions Cards to Produce Fractions

In this activity, students were asked to notate fraction for each of partitioning parts from activity 1. When the students struggled to notate fractions, the teacher tried to provoke the students by posing questions as follows.

Teacher : *Do you remember what is the meaning of fraction notation?*
(…)
Teacher : *Okay, please count how many red cards and blue cards.*
Students : *There are eight red cards and six blue cards.*
Teacher : *So, what is the function of red cards and blue cards?*
Students : *Red cards to write fraction notation of flags and blue cards for water posts.*
Teacher : *Good. Now, we want to write fraction notation for flags. How many flags are there?*
Students : *Eight!*
Teacher : *If this one yarn divided into eight parts, what fraction each part?*
Nanda : ***An eighth**.*
Teacher : *Why do you say it was an eighth, Nanda?*
Nanda : *Ya, because **one part of eight parts means an eighth**.*

From conversation above, it was observed that Nanda and some students who had the same taught with Nanda realized about the concept of fractions as part of a whole. But then, conflict occurred again when they were asked to give fraction notation to each part. One out of six groups came up with the idea of unit fractions while the other groups chose non-unit fractions as fraction notation for each part of flags' and water posts' position as shown in the following figure (figure 36 and 37). The teacher tried to let the students in that group realized by themselves that they would need non-unit fractions to answer next problem in the fourth activity.

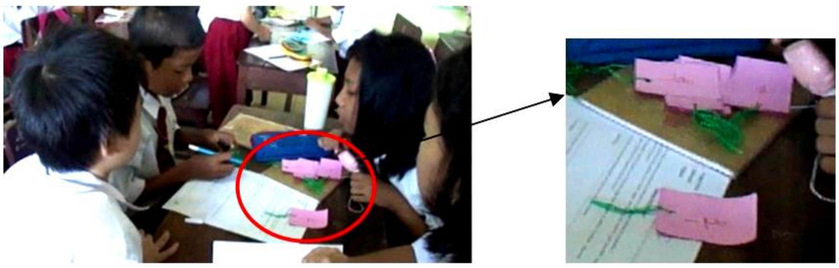


Figure 36. Students Fraction's Notation Using Unit Fractions.

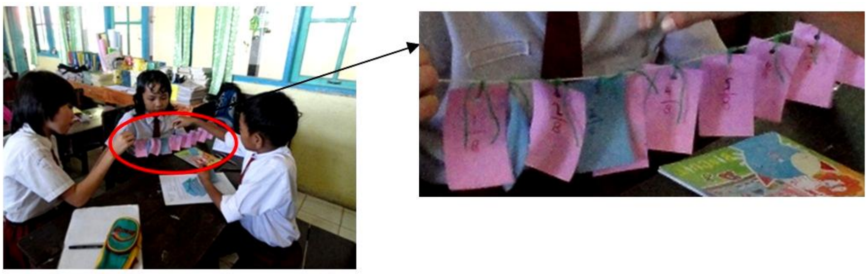


Figure 37. Students Hung Their Non-Unit Fraction Cards on the Yarn.

After students hung the cards on the yarn, they were asked to check their fraction cards positions whether its positions in accordance with the positions of flags and water posts in the map (activity 1). It was observed that most of students realized that there were some fractions in the same position which was also indicated by flags and water posts in the same position as shown in figure 38. It would be discussed in the next meeting when the students shared their findings of their yarn's drawing.

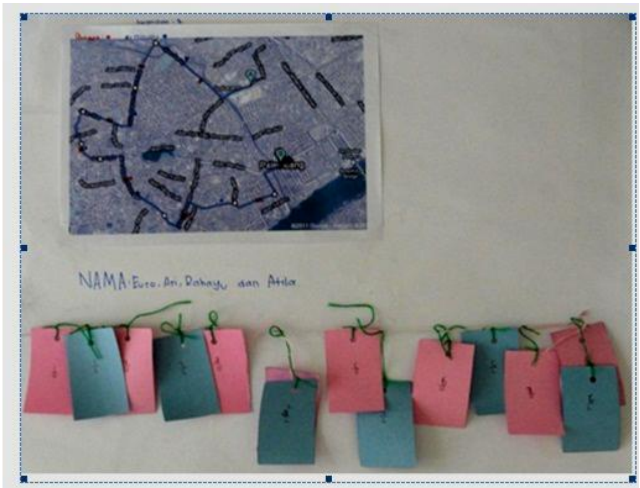


Figure 38. One of Students' Works of Activity 1 and 2.

Answers to Research and Sub Research Questions

In this study, the first activity level which preceded the learning operation in fractions is producing fractions. The data description and interpretation in sub sub chapter 2.a (page 77) and 2.b (page 80) can answer sub research question 1, namely *how does the length measurement activity with the help of yarn provoke students in producing their own fractions?*. The length measurement activity with the help of yarn can provoke students in producing their own fractions. Starting from the activity of '*locating flags and water posts on the running route*', the students were used their informal knowledge of *partitioning* by the help of yarn to measure the total length of the running route. The fractions could be produced as the students were asked to notate the result of partitioning. As reported in sub chapter 2, students came to the idea of an eighth as one part of eight parts when the teacher posed question: '*if this yarn divided into eight parts, what fraction each part?'*. The students then were asked to give fraction notation to each partitioned part. Some students used unit fractions (e.g., $\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}$) and others used non unit fractions (e.g., $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}$). The students have no difficulties in notating fractions since they had already learnt fractions in grade 3. However, the differences of students' answers about unit and non-unit fractions would be realized by the students that they would need non-unit fractions to grasp the next level, namely generating equivalencies.

3. Generating Equivalencies

a. The Number Line Model as a Bridge from Situational Knowledge to the More Formal Knowledge about Fractions

Students' informal knowledge as a result of students' experience in partitioning certain length using tools (e.g., yarn) needed to be developed into more formal knowledge of fractions which would lead to the idea of equivalent fractions. In the next problem, as the *model-of* measuring situation, the students were asked to draw the yarn and the fraction cards in students' worksheet 2. At this moment, this problem could be considered as *referential* when students were initially use tools (i.e., yarn) as a representation of running route. As mentioned in the second tenets of RME, namely the use of models or bridging by vertical instruments, the use of string of yarn served as the base of the *emergence model* of number line as shown in figure 39 below.

Figure 39. One of Students' Drawings as a Representation of Yarn and Fraction Cards.

The use of string of yarn here was as a bridge to the number line model which was in more abstract level. From figure 39, we can see student's initial drawing of representation of string of yarn. In this activity, number line was introduced as a generalization tool of string of yarn.

It was expected that number line could be used as a helpful model to bring the students in enhancing the meaning of fractions, equivalent fractions, comparison fractions, and to solve problem related to fractions and natural numbers. Moreover, we expected that the problems in activity 2 would reveal students' reasoning about fractions in the same position which would lead to the idea of equivalent fractions. As defined by Vanhille & Baroody (2002), if two fractions are equivalent, the ratio between the numerators is the same as the ratio between the denominators. In the math congress, students shared their findings, corrected and criticized other thoughts.

In problem number 4 students' worksheet 2, the students were asked to write down their findings from the drawing of yarn and fraction cards (problem number 3). Most of groups could see that $\frac{3}{6}$ was in the same position with $\frac{4}{8}$ and $\frac{6}{6}$ was in the same position with $\frac{8}{8}$ (figure 40). Moreover, some students could mention the reason why $\frac{6}{6}$ as equal as $\frac{8}{8}$. It is because the value of $\frac{6}{6}$ was 1 nor may the value of $\frac{8}{8}$.

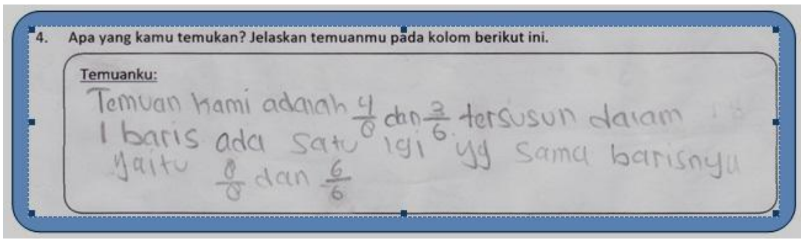


Figure 40. One of Students' Findings from Their Drawing of Yarn and Fraction Cards.

The translation of figure 40 is as follows.

Our findings is $\frac{4}{8}$ *and* $\frac{3}{6}$ *arranged in one row. There is one more which in the same row. It is* $\frac{8}{8}$ *and* $\frac{6}{6}$*.*

From students' answer above, we can see students' familiar language in defining the idea of equivalent fractions. They mentioned the words *arranged in one row* to indicate fractions in the same position. This idea would be discussed further in the math congress 1.

b. Math Congress 1: Communicating and Developing Ideas about Fractions on the Number Line

Students' thought about fractions in the same position was brought into a discussion in the math congress. This math congress was conducted to facilitate and develop students' acquisition of the use number line as a helpful model to bring them to the idea of the meaning of fractions and equivalent fractions. The benefit gained from math congress was not merely communicating a student's idea, but also stimulating other students to develop various strategies.

Considering the importance and the use of students' drawing of yarn and fraction cards, the teacher started the discussion by inviting two groups which had different drawing of yarn to present their answers on the board (figure 41). This is one of ways to emphasize the communication of the problem because students occasionally ignored an oral problem. The combination of oral and written problem will engage students in more active thinking and discussion.

Figure 41. Two Groups Presented Their Drawings on the Board.

After two groups finished writing their solutions on the board, the teacher tried to engage all students in the discussion. The teacher challenged the students to find out the differences between Beladas's and Kucing's drawing of yarn (figure 42).

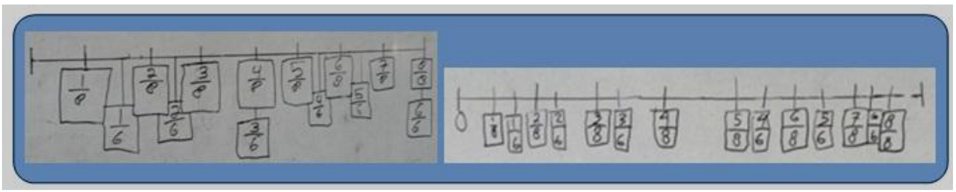


Figure 42. From Left to Right: Beladas's and Kucing's Drawings of Yarn and Fraction Cards.

Teacher : *Now we have two drawings from Beladas and Kucing groups. Could you find something different from the drawings of Beladas and Kucing?*

(...)

Teacher : *Okay, now we start from Rajawali group. What is the difference from the picture of this group* (pointing Beladas' s drawing) *and this picture* (pointing Kucing's drawing). *If the pictures are too small, you are allowed to come forward.*

One of students in Rajawali group came forward.

Pandu : *Mmm..in Beladas' drawing,* $\frac{8}{8}$ *and* $\frac{6}{6}$ *are in the same point, while in Kucing's drawing are not in the same point.*

Teacher : *Good job. Okay, any other arguments?*

Viola : *Ya, I have.* $\frac{4}{8}$ *and* $\frac{3}{6}$ *are in the same point, while in Kucing's drawing* $\frac{4}{8}$ *and* $\frac{3}{6}$ *are not in the same point* (figure 43).

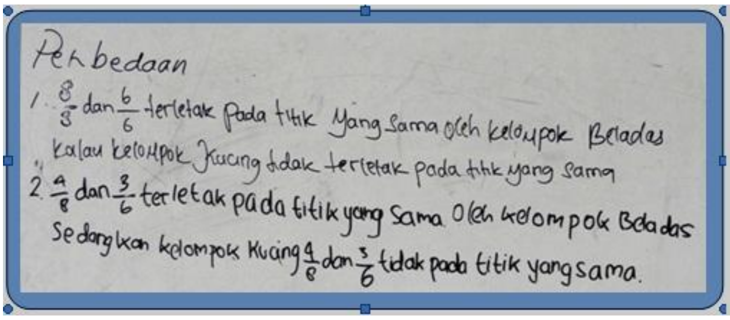


Figure 43. Students' Arguments about the Differences of Beladas's and Kucing's Drawing of Yarn.

After all students thought that there was no other differences between Beladas's and Kucing's drawing, then the conversation continued as follows.

Teacher	:	*Can you explain why* $\frac{8}{8}$ *and* $\frac{6}{6}$ *and also* $\frac{4}{8}$ *and* $\frac{3}{6}$ *are in the same point?*
Dhea	:	*Because it* ***has the same value****.*
Teacher	:	*Hmm..what do you mean by the same value?*
Dhea	:	*Ya, because the value of* $\frac{8}{8}$ *was 1 and also the value of* $\frac{6}{6}$ *was 1.*
Teacher	:	*How about* $\frac{4}{8}$ *and* $\frac{3}{6}$*? Do these fractions also have the same value? Can you proof it?*
Tirta	:	*Ya, it can be simplified.* (Tirta showed his simplification of fractions on the whiteboard as shown in figure 44)

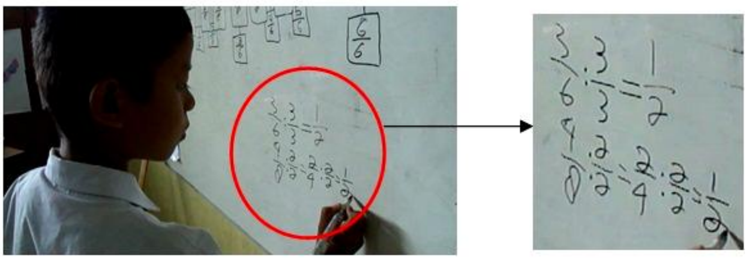


Figure 44. Tirta's Idea about Simplification of Fractions.

From conversation and pictures above, it was observed that Tirta came up with the idea of simplifying fractions which could produce the same value of fractions. The teacher introduced the term of *equivalent fractions* meant by fractions in the same position which have the same value. At this phase, the students were able to construct their multiplicative reasoning of fractions within equivalent fractions. The mathematical idea underlay this phase was to maintain equivalence, the ratio of the related number must be kept constant.

c. Introducing the Concept of Multiplication as Repeated Addition by using Jumping on the Number Line of Fractions

From the drawing of yarn and fractions cards represented by a number line, it can be explored more about equivalencies which can lead to the concept of multiplication of fractions as repeated addition of fractions. The word '*jumps'* was used as a word that could provoke the students to get the meaning of that concept. It was started by providing problem: '*how many* $\frac{1}{8}$*- jumps from zero point to* $\frac{5}{8}$*?*'. Most of the students could answer that there were 5 times of $\frac{1}{8}$ -jumps from zero point to point $\frac{5}{8}$.

The problem continued by providing problem about multiplication in natural number, namely 2×3. The students were asked to explain the meaning of 2×3. It was expected that the students came up with the idea multiplication as repeated addition. In fact, the students still confused about the correct meaning of 2×3, whether it was $3+3$ or $2+2+2$. Some students answered it was $3+3$ and the other students answered $2+2+2$. The teacher tried to remind the students about the words '*two times adding 3*', then the students realized, the answer should be $3+3$. To strengthen students' concepts about multiplication as repeated addition, the teacher gave another multiplication problem. This concept will be used when the students were asked to find the relation between $\frac{1}{8}$-jumps with point $\frac{5}{8}$.

In the next problem, students were asked to explain the relation between $\frac{1}{8}$-jumps with point $\frac{5}{8}$. It was expected that the students would relate the idea of multiplication as repeated addition when they found there were five times of $\frac{1}{8}$-jumps to reach point $\frac{5}{8}$. Ari shared his thought about jumps. He said it can be written as an addition, because it jumped five times. In the figure 45, we can see Ari's thinking about the word 'jumps' and his transition from familiar word to the mathematical notation '+'.

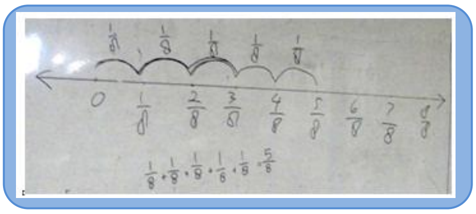


Figure 45. Ari's Thinking about '*Jumps*'

Ari's finding about adding fractions was responded by Ihsan. He related the addition of fractions with multiplication of fractions as shown in figure 46.

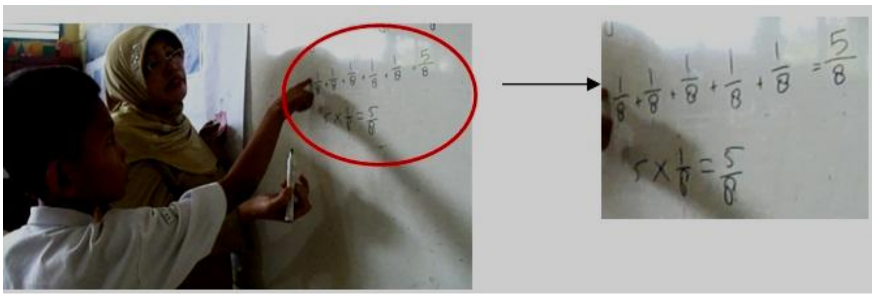


Figure 46. Ihsan's Thought about the Relation Between Addition of Fractions with Multiplication of Fractions.

He explained that $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ is the same as $5\times\frac{1}{8}$. The other students agreed Ihsan's idea because they had discussed about the concept of multiplication as repeated addition. Therefore, they used their knowledge about multiplication as repeated addition to come to the idea of multiplication natural numbers by fractions as repeated addition of fractions.

Answers to Research and Sub Research Questions

The idea of number line appeared when the students were asked to draw the representation of yarn and fractions cards hung on it in activity 2 as described in sub chapter 3a. This fact contributed for an answer for the second sub research question, namely *how does the string of yarn lead the students to the idea of number line?*. Due to the form of yarn which is thin, it led the students to draw a line as representation of a string of yarn. This line later named as number line. Moreover, as described in sub chapter A5 (page 59), this number line was called as number line of fractions when students realized the existence of fractions in the number line. Connected to the second tenet of RME, namely the use of models or bridging by vertical instruments, the use of string of yarn here served as the base of the *emergence model* of number line. The representation of the string of yarn became the *model of* measuring situation.

Moreover, this part of analysis also answer sub research question (3), namely *how does a number line lead the students in generating equivalencies which then lead them to the idea of multiplication of natural numbers by fractions?*. The number line was proven as a powerful model to encourage the students in generating equivalencies of fractions. Through generating equivalencies, the students could relate equivalent fractions and the relation among fractions with the idea of multiplication of fractions.

As stated in VanHille & Baroody (2002), teacher could focus students' attention on multiplicative reasoning of fractions as they taught equivalent fractions. If two fractions were equivalent, the ratio between the numerators was the same as the ratio between the denominators. As described in sub chapter 3a (page 83), from students' drawing of number line as a representation of yarn (problem 3 in activity 2), it was found there were two pairs of fractions which were in the same position, namely $\frac{3}{6}$ with $\frac{4}{8}$ and $\frac{6}{6}$ with $\frac{8}{8}$. To proof these two pairs of fractions equal, the idea of simplifying fractions was used. At this phase, the students developed their multiplicative reasoning of fractions through equivalent fractions which can be seen from fractions in the same position on the number line.

Moreover, the number line also led the students in learning multiplication of fractions when they were asked to find the relation between fractions. As described in sub chapter 3c (page 88), the use of jumping on the number line of fractions provoked students to the idea of multiplication of fractions as repeated addition of fractions. For instance, through the problem of finding the relation between $\frac{1}{8}$-jumps and $\frac{5}{8}$ on the number line, students could see that there were five jumps of $\frac{1}{8}$-jumps from zero point to $\frac{5}{8}$. Then they related this with the definition of multiplication as repeated addition. Furthermore, it was written in more formal mathematical notation as $5 \times \frac{1}{8}$.

4. Operating through Mediating Quantity

Based on the explanation in the third activity level, to lead to the idea of fractions as operator, we involved the length to a given unit. Through the activity of '*determining who is running farther*', it was expected that the students could compare fractions within a certain length and informally use fractions as multipliers. The fraction which at the first was described as part of a whole relationship now became a fraction as operator.

a. The activity of '*Determining Who is Running Farther*' as the Beginning of the Involvement of Fractions as Operator

As an opening of the activity, the teacher started an initial discussion about the story in students' worksheet 4. It was observed that the students did not have any difficulties in translating the position of Bimo who stopped on the fifth flag into fractions which meant

$\frac{5}{8}$ and also the position of Ari who stopped on the fourth water post which meant $\frac{4}{6}$. However, after some times, the students still struggled in translating the question, '*how many kilometers have Bimo and Ari run?'*. They could not relate the question into the concept of fractions multiplication. For instance, they knew the position of Bimo was $\frac{5}{8}$ but they did not know how to find $\frac{5}{8}$ of 6 kilometers.

In this situation, the teacher tried to provoke the students by introducing the use of double number line. She gave a short clue about the use of double number line where they could put the distance (6 kilometers) above the number line and fractions below the number line as shown in figure 47. She took example of finding Bimo's distance when he stopped on the fifth flag.

Figure 47. Teacher Tried to Introduce the Use of Double Number Line.

Teacher attempted to connect this double number line model to students' strategies in partitioning the route into eight by proposing question, '*can you find a half of the distance in the number line?*'. This kind of guidance from the teacher and students' reaction to this guidance showed that the string of partitioned yarn played an important role as a bridge from students' informal knowledge of partitioning to more formal mathematical thinking through double number line model.

Most students had no difficulties when they were asked to find a half of the distance, which was 3 kilometers. They got from dividing 6 kilometers by two. A problem occurred when the students were asked to find $\frac{5}{8}$ of 6 kilometers.

When there was no student who realized that they need to find '*how many kilometers in each part* ($\frac{1}{8}$*)*, the teacher asked the students to give attention back to her drawing of double number line. Suddenly, Ferdi realized that they only need to find certain length from $\frac{4}{8}$ to $\frac{5}{8}$ because Bimo had run 3 kilometers which was a half way of the distance. All students agreed that they need to find the length of $\frac{1}{8}$. There were two strategies used by the students when finding the length of $\frac{1}{8}$. These strategies are described as follows.

First Strategy

The first strategy of students was finding a half of a half of the whole distance. The following excerpt is an example of Kucing group who used this strategy.

Researcher	:	*How did you find the length of* $\frac{1}{8}$ *?*
Rio	:	*Because we have known a half of the distance which is in the point* $\frac{4}{8}$*. To find the length of* $\frac{1}{8}$ *which is a half of* $\frac{2}{8}$*, firstly, we need to find the length of a half of* $\frac{4}{8}$*.*
Ferdi	:	*The length from zero point to* $\frac{2}{8}$ *is* $1\frac{1}{2}$ *kilometers* (as shown in figure 48).
Researcher	:	*Now, what did you do next?*
Rio	:	*Finding the length of* $\frac{1}{8}$ *which by dividing* $1\frac{1}{2}$ *kilometers by two.*

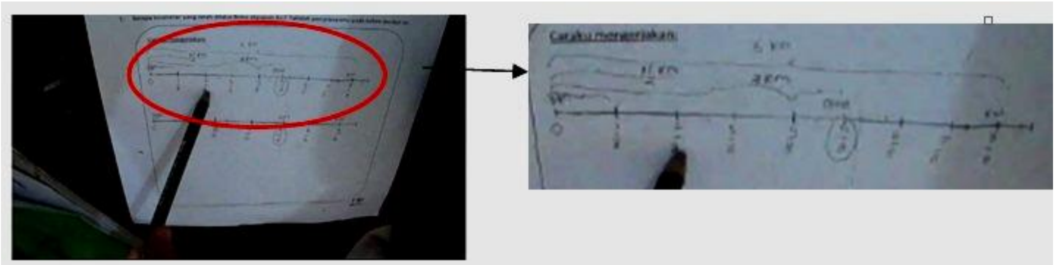


Figure 48. Kucing Group’s Strategy about a Half of a Half of the Whole.

A conjecture is derived from this strategy namely the students connected the way they found the length of $\frac{1}{8}$ to the way they partitioned yarn. After they found the length of $\frac{2}{8}$, this group had difficulties when they need to divide $1\frac{1}{2}$ kilometers by two to find the length of $\frac{1}{8}$. After a while, the students used the idea of fair sharing to divide $1\frac{1}{2}$ kilometers by two described as follows (figure 49).

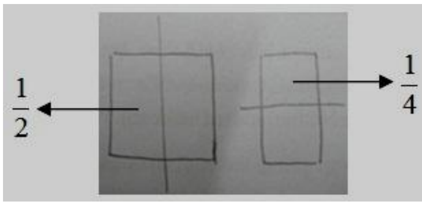


Figure 49. Students’ Way in Dividing $1\frac{1}{2}$ by Two.

1. First, they draw picture which represented one and a half. They called this picture was cake. They wanted to share this cake for two people.
2. Then they divided each picture into two.
3. So, each person got a half and a fourth which meant three-quarter ($\frac{3}{4}$)

From this group’s work, it was obtained that $\frac{3}{4}$ kilometers was the length of $\frac{1}{8}$.

Second Strategy

The following excerpt shows another strategy used by Beladas group when finding the length of $\frac{1}{8}$ (figure 50).

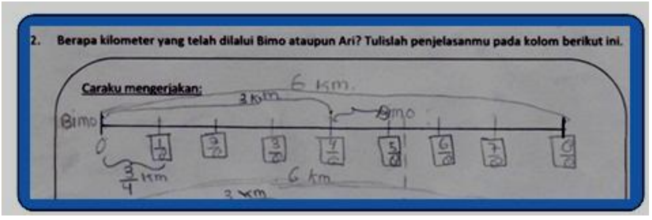


Figure 50. Beladas Group's Idea about Finding the Length of $\frac{1}{8}$

Teacher	:	*I saw you used different way from that group and the others. Could you explain the way you find the length of* $\frac{1}{8}$ *?*
Euro	:	*Mm..because the length from zero point to* $\frac{4}{8}$ *is 3 kilometers, and there are 4 parts from zero point to* $\frac{4}{8}$*, so we only divide 3 by 4 which gets* $\frac{3}{4}$*.*

The phrase '*we only divide 3 by 4 which gets* $\frac{3}{4}$' shows that Euro used his knowledge about dividing certain length by the number of parts.

From two strategies above, the students continued finding $\frac{5}{8}$ of 6 kilometers. Most of groups answered the problem by adding fractions $\frac{3}{4}$ as many as five times. Moreover, the students could relate this repeated addition of fractions to multiplication of natural number by fraction as shown in figure 51. This strategy matches to the conjecture that was formulated in chapter 4, namely the students used repeated addition with $\frac{3}{4}$ as an object which they wanted to add as many as five times.

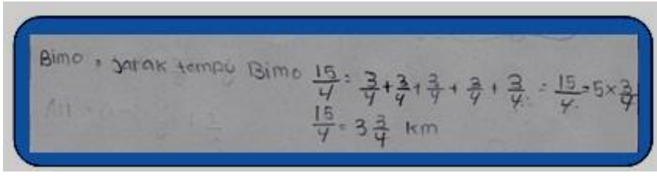


Figure 51. Student's Answer of the Problem $\frac{5}{8}$ of 6 Kilometers.

There was a surprising moment occurred when interviewed was held during classroom activity. Ferdi and Euro said that there was another way to find $\frac{5}{8}$ of 6 kilometers. They just only needed to add 3 kilometers with the length of one $\frac{1}{8}$-jumps as shown in figure 52.

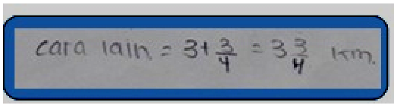


Figure 52. Another Way to Find $\frac{5}{8}$ of 6 Kilometers.

Besides finding the distance of Bimo, the students also needed to find the distance of Ari which stopped in the fourth flag. They need to find $\frac{4}{6}$ of 6 kilometers. We found out that there was a group who came up with a surprising answer. Besides using double number line model, they used different way as shown in figure 53.

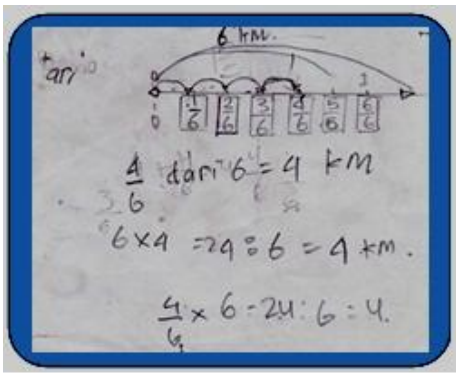


Figure 53. Spenta Mania Group in Solving the Problem of $\frac{4}{6}$ of 6 Kilometers.

From the answer above, we asked the group to share their strategy of solving the problem of $\frac{4}{6}$ of 6 kilometers. It was known that this idea came from Dhea, one of students in Spenta Mania group.

Researcher	:	*Dhea, could you explain how did you get 4 kilometers as the answer of* $\frac{4}{6}$ *of 6 kilometer?*
Dhea	:	*I multiplied 6 by 4* (pointing 6 besides the word 'dari' and 4 (the numerator of $\frac{4}{6}$)) *and the result was 24. Then I divided it by 6* (pointing 6 as denominator of $\frac{4}{6}$).
Researcher	:	*Can you explain how did you get that idea?*
Dhea	:	*When we learnt about 'jumps', there were 5 jumps from zero point to* $\frac{5}{8}$. *We then related it to the idea of multiplication, it meant* $5\times\frac{1}{8}$. *If we see, when we multiplied 5 by 1 then the result is 5, the numerator of fraction* $\frac{5}{8}$. *Then we only need to write denominator of* $\frac{1}{8}$, *which is 8 as denominator of the result. It was obvious that* $5\times\frac{1}{8}=\frac{5}{8}$.

From the fragment, it can be seen that Dhea came up with the rules for operation with fraction. She found the rule by herself when she found the connection between $5\times\frac{1}{8}$ and $\frac{5}{8}$. Then she used her idea to find the result $\frac{4}{6}$ of 6 kilometers. Dhea's idea will be discussed further in fourth level, namely doing one's own production.

Answers to Research and Sub Research Questions

The description and interpretation above can answer sub research question (4), namely *how does the involvement of certain length as the mediating quantity lead the students to the idea of multiplication of fractions by natural numbers?*. The involvement of certain length as the mediating quantity does lead the students to the idea of multiplication of fractions by natural numbers. Fractions, which had at first been used to describe section of the route, were now applied to the distance of the running route. It had been promoted to being fractions in the operators.

Continuing from the number line which had emerged from the previous activity as a model of measuring situation, in the activity of '*determining who is running farther*', a natural number was involved on the number line as the distance of the running route. Fractions on the number line transformed from fractions as part of a whole into fractions as operator. By connecting students' strategies in partitioning the route, the use of double number line could be introduced as a helpful model to find the length of certain fractions.

As described in sub chapter 4 (page 91), the students came to the idea of multiplication of fractions by natural numbers when they were asked to find $\frac{5}{8}$ of 6 kilometers and $\frac{4}{6}$ of 6 kilometers. For instance, we took the problem of $\frac{5}{8}$ of 6 kilometers. The idea of multiplication of fractions as repeated addition of fractions appeared when the students added the length of $\frac{1}{8}$ which was $\frac{3}{4}$ kilometers as many as five times because it took five jumps from zero point to $\frac{5}{8}$. Then it was written into a more formal mathematics operation as $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}$. From this repeated addition of fractions, the students then related it with the idea of multiplication natural numbers by fractions which then was written as $5\times\frac{3}{4}$. In addition to students success in linking the issue with the idea of multiplication natural numbers by fractions, through discussing Spenta Mania's strategy in solving problem $\frac{4}{6}$ of 6 kilometers (see page 96) which then was written in mathematical notation as $\frac{4}{6}\times 6$, the students began to transform the word '*of*' into mathematical notation '$\times$'. Spenta Mania group succeeded in linking the problem of finding the length of certain fractions with multiplication fractions by natural numbers. It also influenced other students to come up with the idea of multiplication of fractions by natural numbers through the math congresses as the moments of learning where the educational principle of interaction was applied.

5. Doing One's Own Production

At this level, progression meant that the students were able to solve problems in a more and more refined manner at the symbolic level. As mentioned in the third tenet of RME, the biggest contributions to the learning process were coming from *student's own creations and contributions* which led them from their own informal to the more standard formal methods. Students' strategies and solutions could be used to develop the next learning process.

a. Math Congress 2: Communicating and Developing Ideas

As a continuation of '*determining who is running farther activity*', math congress was held in the next meeting. In this math congress (activity 5), teacher tried to bring the students into a discussion by asking two groups which had different strategies (figure 54) to present their solution on the board. The groups were Singa Mania and Spenta Mania groups. They were asked to write down their strategy in finding the distance of Ari.

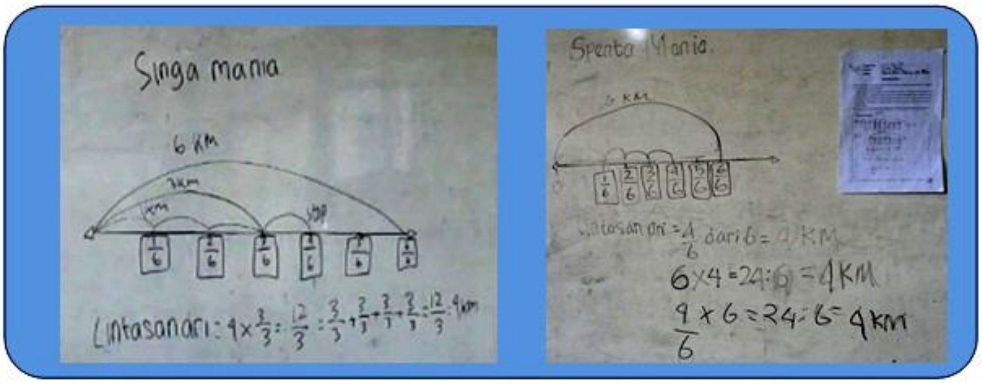


Figure 54. Singa Mania's and Spenta Mania's Strategies in Finding the Distance of Ari.

The teacher opened the discussion by reminding the students about the problem of Ari's distance. The representative of Singa Mania, Viola, tried to explain how they got $4\times\frac{3}{3}$. They got 4 from the number of jumps from the starting point to the point where Ari stopped (i.e., $\frac{4}{6}$). They got $\frac{3}{3}$ from 3 kilometers divided by 3 parts (from starting point to point $\frac{3}{6}$). The result was $\frac{12}{3}$ which described again as repeated addition of $\frac{3}{3}$ five times, or can be written as $\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}$. In this phase students started to use double number line model as a *model for* more mathematical reasoning.

The discussion continued by exploring Spenta Mania's solution. From the conversation with Dhea (page 97), one of students in Spenta Mania group, she connected the result of $5\times\frac{1}{8}$ which produced $\frac{5}{8}$. She used the idea of '*jumps*' to come to the idea of multiplication of fractions as repeated addition. The *model-for* more mathematical reasoning in this phase was the reasoning of relationship among fractions i.e. the equivalent fractions. She concluded that to multiply 5 by $\frac{1}{8}$, she only needed to multiply 5 by 1 then divided it by 8 (the denominator of $\frac{1}{8}$). Therefore, by using the same strategy, she found the answer of $\frac{4}{6}$ of 6 kilometers by multiplying 4 by 6 kilometers then divided it by 6 (the denominator of $\frac{4}{6}$).

Spenta Mania also came up with the idea of translating $\frac{4}{6}$ of 6 as $\frac{4}{6}\times 6$. They realized that to find $\frac{4}{6}$ of 6, they did multiplication of fractions where the function of fraction was as operator. To strengthen students' understanding, teacher tried to give another example namely $\frac{2}{5}$ of 10. Firstly, she asked the meaning of the problem as described in the following fragment.

Teacher	:	*Do you know what is the meaning of* $\frac{2}{5}$ *?*
Students	:	*The point where Bimo stops.*
Teacher	:	*Okay, you said* $\frac{2}{5}$ *is the point where Bimo stop. Now, what is 2?*
Students	:	*The second flag where Bimo stops.*
Teacher	:	*And what about 5?*
Students	:	*The number of the flags.*
Teacher	:	*What is 10?*
Students	:	*The length of the running track.*

From the fragment, it can be seen that the students still connect the problem with Bimo who run $\frac{2}{5}$ of the total length of the running route. However, they did not use double number line again to solve this problem. They preferred to use Dhea's idea to solve problem which they only need to multiply 2 by 10 then divide it by 5. The use of student's contribution as the *model for* more formal reasoning showed that *general level* of modeling has been attained by the students.

6. On the Way to Rules for the Operation with Fractions

In the formal level students' reasoning with conventional symbolizations started to be independent from the support of models for mathematical activity. The last level of emergent modeling, the formal level, the focus of discussion move to more specific characteristics of models related to the concept of equivalent of fractions and multiplication fraction with natural numbers.

Throughout mini lesson (activity 6) which included fractions as operator, the students reflected on the rule for the multiplication fractions with natural numbers. The transition to a more formal fractions was preceded by stimulating students to contribute their own informal ways of working which led by Dhea's idea about her rule in multiplying fractions by natural numbers. In this activity the students were asked to make their own story related to the problem (figure 55).

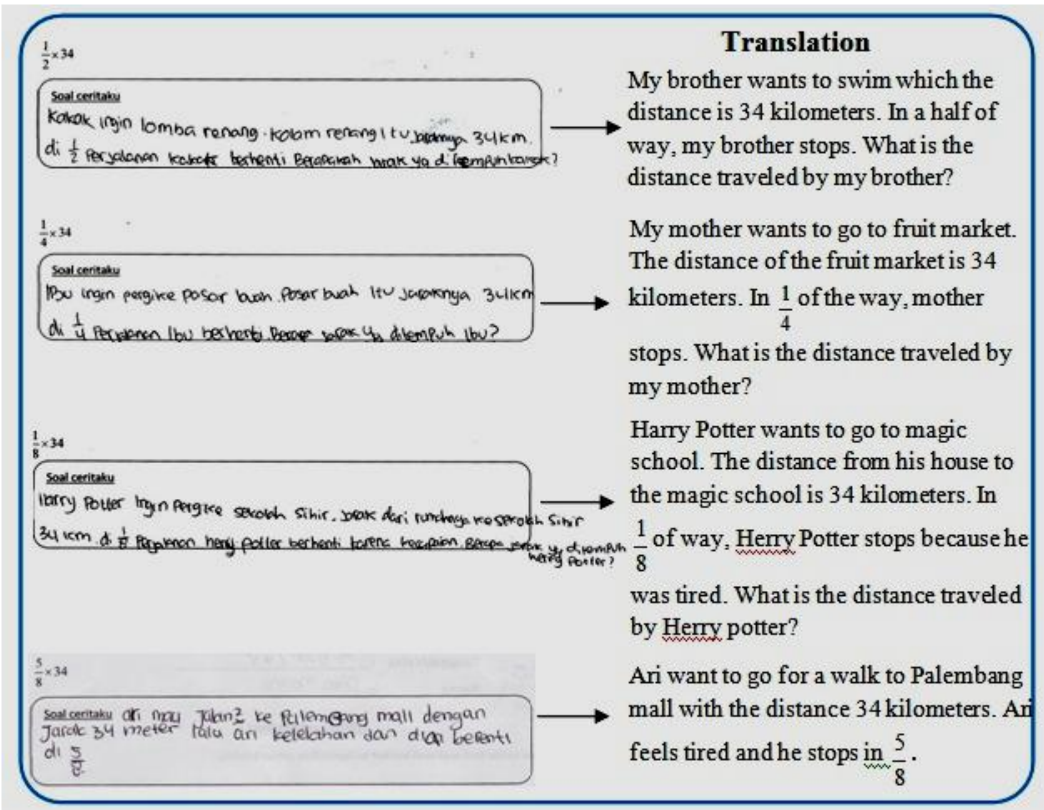


Figure 55. Nanda's Word Problem on the String Number in Mini Lesson.

From Nanda's word problem above, it seemed that she did not notice about the meaning of distance which indicates the length from one place to another. In three out of four problems, she only wrote, for instance, 'my brother wants to swim which the distance is 34 kilometers'. Only in problem 3 she included 'from his house' in her word problem to indicate that the distance was from Harry Potter's house to the magic school. She directly used fraction in the word problem which indicated the place where the subjects of the stories stop. This moment was in line with our conjectures, namely students still tried to relate the problem with the running problem.

Most students used Dhea's strategy to solve the problems. Figure 56 was one of students' strategies using Dhea's strategy. However, there was a student, namely Jessica, who still use double number line model in solving problem shown in figure 57.

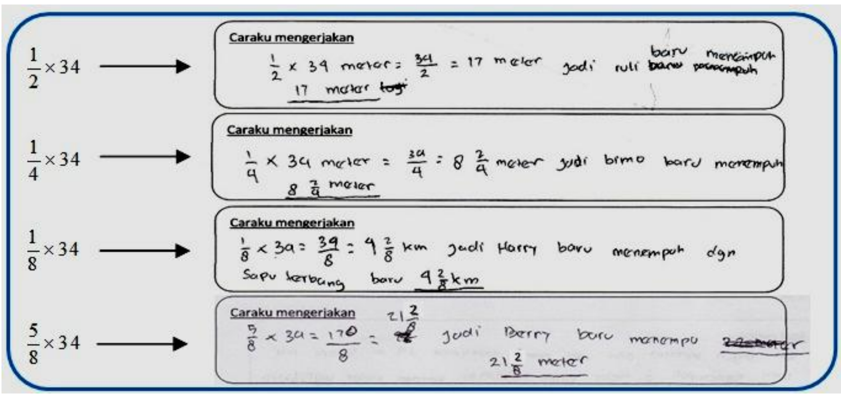


Figure 56. Rio's Strategy in Mini Lesson.

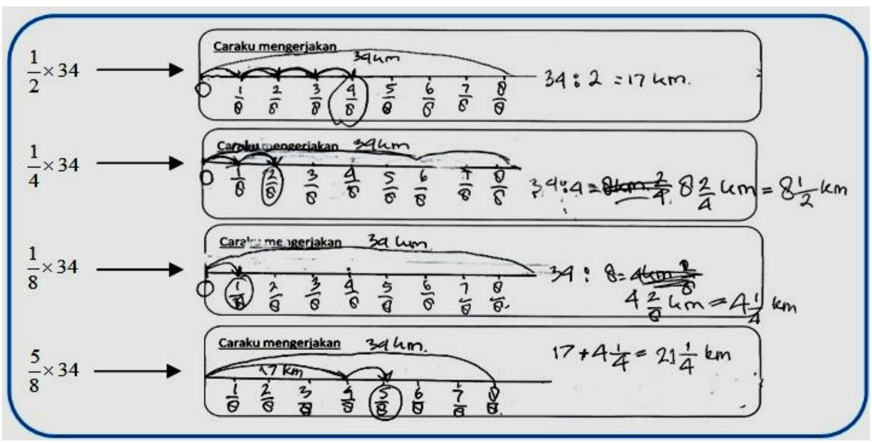


Figure 57. Jessica's strategy in Mini Lesson

As we can see from Jessica's answers above, she used double number line model as a helpful tool to solve problems which include fractions as operator. It was conjectured that double number line model helped her to develop her conceptual understanding of fractions as numbers. However, she combined her double number line with Dhea's idea about the rule of operation with fractions.

It was also conjectured that she gave reasoning with fractions by using the relation among fractions. It can be seen from her answer of $\frac{5}{8}\times 34$. She added 17, as a half of the running route, with $4\frac{1}{4}$, as the length of $\frac{1}{8}$ part. This strategy matches to the conjecture that was formulated in chapter 4, namely decomposing $\frac{5}{8}$ into $\frac{4}{8}$ plus $\frac{1}{8}$. The idea of equivalent fractions was used to transform $\frac{4}{8}$ into $\frac{1}{2}$. Moreover, this strategy can be connected to the more formal mathematics which we called as the *distributive property* which holds for multiplication over addition for fractions. It can be written as follows.

$$\left(\frac{1}{2}+\frac{1}{8}\right)\times 34=\left(\frac{1}{2}\times 34\right)+\left(\frac{1}{8}\times 34\right)=17+4\frac{1}{4}=21\frac{1}{4}\text{ kilometers}$$

At this moment, the idea of distributive strategy was not discussed further because of the limitation of time. The result of mini lesson informed that there were merely about 72,35% of the students seemed to correctly answer problems in the mini lesson. So, it is conjectured that the students still need to do more practices of operation with fractions in particular multiplication fractions with natural numbers.

Answers to Research and Sub Research Questions

The data description and interpretation in sub sub chapter 5 (page 99) and sub sub chapter 6 (page 102) can answer sub research question (5), namely *how can one's own production about the idea of multiplication of fractions lead students on the way to rules for multiplying fractions by natural numbers?*. At the last two levels, namely doing one's own production and on the way to rules for operation in fractions, progression meant that the students were able to solve problems in a more and more refined manner at the symbolic level. As mentioned in the third tenet of RME, the biggest contributions to the learning process were coming from *student's own creations and contributions* which led them from their own informal to the more standard formal methods.

By bringing Dhea's idea to the math congress, the classroom community came to a taken-as-shared way of performing the more efficient strategy in solving problem about multiplication of fractions (described in sub sub chapter 5 (page 99)). Most students preferred to use Dhea's strategy as described in sub sub chapter 5. They agreed that Dhea's strategy is more efficient than double number line model. Dhea had made a significant contribution to the classroom community for the idea that to multiply fraction with natural number, they only needed to multiply the numerator with the given natural number and put the denominator below the fraction result. She came up with the rule of multiplication fractions by natural numbers when she saw the connection between $5\times\frac{1}{8}$ and $\frac{5}{8}$. In this respect, the teacher played an important role in enhancing other students' thinking. Before approving Dhea's idea, she asked all students to compare Singa Mania's strategy which used double number line model to solve problem $\frac{4}{6}$ of 6 kilometers and Spenta Mania's strategy which use Dhea's strategy (see page 97). They realized that by using different strategy, it produced the same result. Therefore, they conclude that Dhea's idea was more efficient since Dhea had her argument how she came up to the rule of multiplication of fraction by natural number. This phase showed that student's own production lead other students on the way to rules for multiplying fractions by natural numbers.

7. The Post-Assessment of Students' Learning Operation with Fractions

The post-test was given to all students in order to measure students' understanding after they followed the learning activities. If they were successful in solving the task in the post-test, their work indicates an understanding in progress. We will not fully describe students' answers, but we will consider students' reasoning in doing this post-test.

There were four problems with sub problems in it. The problems were related to the sequence of activities. There was one problem which connected with problem in pre-test, namely partitioning certain object (problem 1).

The first problem was about partitioning two ropes into eight and twelfth parts equally. The goal of this problem was to check students' capability in partitioning certain length as they had worked on the first activity in the sequence of activities. Most students could partition the rope correctly. They had no difficulties in partitioning rope into eight parts equally (problem 1.a). It might because they already had experience in doing it. There were some students who still struggled in partitioning the rope into twelfth parts equally (problem 1.b). However, about 75% of the students succeeded in partitioning the rope. One of them was Dhea. Below is shown Dhea's work (figure 58).

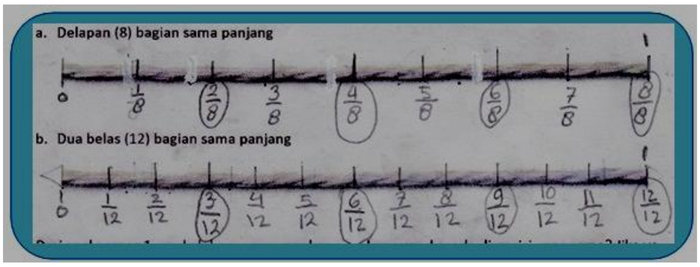


Figure 58. Dhea's Work in Partitioning Rope.

In Dhea's work, there were some fractions which were circled. These signs were to answer the next problem about fraction in the same position. This problem was to check students' knowledge about equivalent fraction. Students who partitioned the rope correctly in the first problem had no difficulties in finding fractions in the same position. The example of students' mistake in positioning the fractions is shown in figure 59.

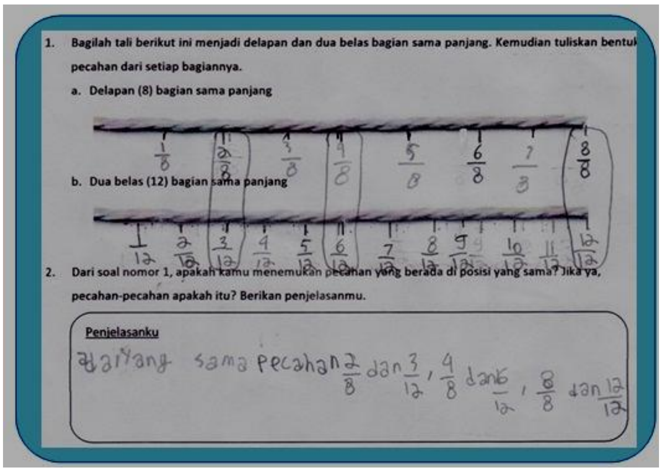


Figure 59. Dinda's Work of Partitioning the Rope and Finding Fractions in the Same Position.

From the picture above, Dinda's work above, we could see that she erased her correct position of fraction $\frac{9}{12}$ then changed the position toward the left side. It could be due to several factors. Probably she looked at her friend's answer which only got three pairs of fractions in the same position which are $\frac{2}{8}$ and $\frac{3}{12}$, $\frac{4}{8}$ and $\frac{6}{12}$, $\frac{8}{8}$ and $\frac{12}{12}$. This event often occurs in Indonesian classroom culture. Students often lack of confidence of their answers. However, there were also some students who could answer the problem correctly. One of them was Tri. Below is shown Tri's work (figure 60). We assume the students who could answer the problem correctly remembered the activity of describing the relations among fractions activity (activity 2).

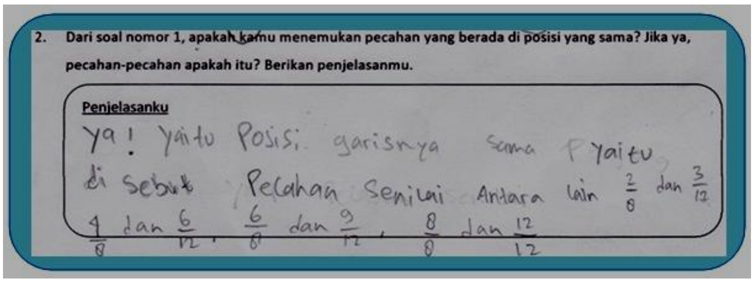


Figure 60. Tri's Work in Finding Fractions in the Same Position.

The translation of Tri's work is as follows.

Ya! The same line position called as equivalent fractions which are $\frac{2}{8}$ *and* $\frac{3}{12}$, $\frac{4}{8}$ *and* $\frac{6}{12}$, $\frac{6}{8}$ *and* $\frac{9}{12}$, $\frac{8}{8}$ *and* $\frac{12}{12}$.

Next problem about the relation between fractions $\frac{1}{10}$ and $\frac{5}{10}$. The goal of this problem was to check students' knowledge about the idea of '*jumps*' which would lead them to the idea of multiplication of fractions as repeated addition of fractions.

Most students had no difficulties in completing fractions in the number line of fractions because it arranged from least to greatest fractions and there were some fractions as a clue. In the following is one of students' answer problem number 3 (figure 61).

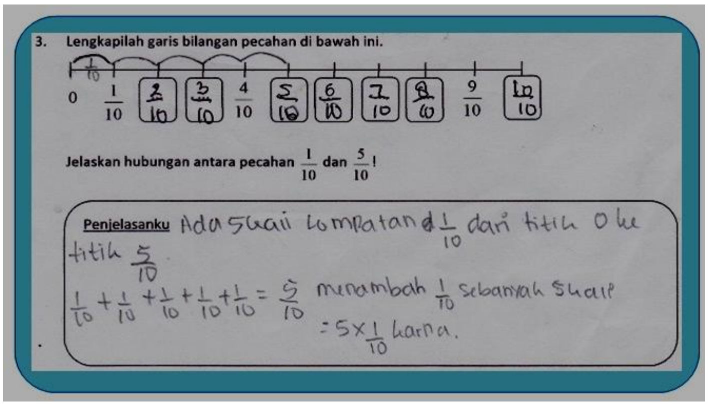


Figure 61. Jessica's Answer about the Relation Between Fractions $\frac{1}{10}$ and $\frac{5}{10}$.

The translation of Jessica's work is as follows.

There are five $\frac{1}{10}$ *- jumps from 0 point to* $\frac{5}{10}$ *point.*
$\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}=\frac{5}{10}$ *add* $\frac{1}{10}$ *as much as 5 times* $5\times\frac{1}{10}$.

Jessica wrote $\frac{1}{10}$ between 0 and $\frac{1}{10}$ in the number line of fractions which indicated a jump of $\frac{1}{10}$. She used the idea of '*jumps*' to describe $\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}=\frac{5}{10}$. From her answer, we could see that she came up with the idea of multiplication natural number by fraction using her own words which was '*add* $\frac{1}{10}$ *as many as 5 times'.*

The fourth problem was exactly the same as the string of number in mini lesson (activity 6). Students were asked to make their own word problems then solve the operation with fractions. About 88,23% students used Dhea's idea about the rule of operation with fractions. It means that the students had came to the level of formal mathematical reasoning which was no longer dependent on the support *model for* mathematical activity. One of them is shown in figure 62.

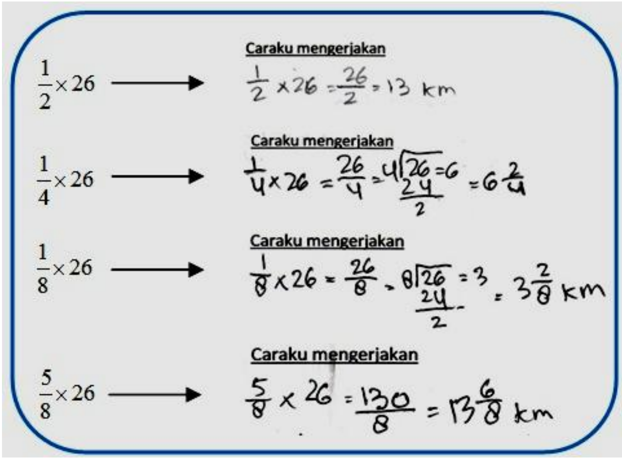


Figure 62. Nanda’s Answers about Multiplication Fractions with Natural Numbers.

From Nanda’s answer above, there was a miscalculation in problem of $\frac{5}{8}\times 34$. The answer should be $16\frac{2}{8}$ or $16\frac{1}{4}$. In this problem, 65,63% of the students could solve all string of numbers correctly, 25% of the students had miscalculated, and the rest still struggled in solving the problems.

There were two students who still used double number line model as a helpful tool to solve problem number 4, namely Jessica and Rizki. In the following is Jessica’s answer of problem number 4 using double number line model (figure 63).

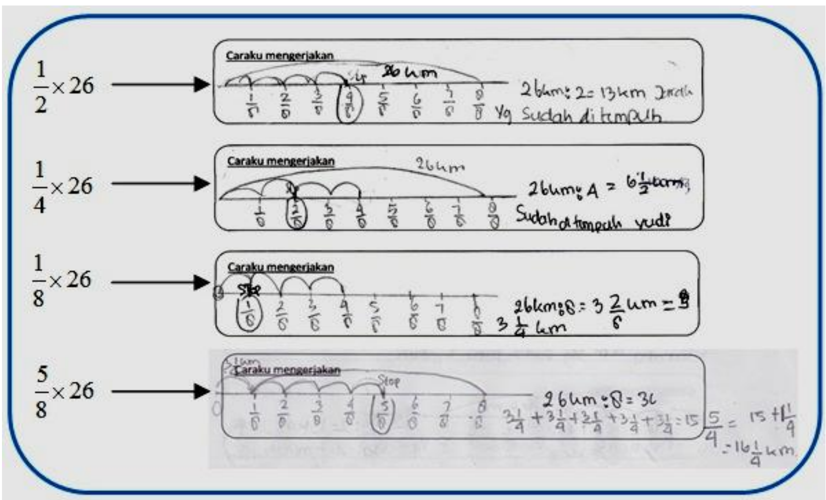


Figure 63. Jessica’s Work of Problem Number 4.

From Jessica's answer above, we assume that the double number line model helped her to make partial product ($\frac{1}{2}\times 34$, $\frac{1}{4}\times 34$ and $\frac{1}{8}\times 34$) to determine the product of the whole ($\frac{5}{8}\times 34$). She used the same strategy as she worked on the Mini lesson in activity 6.

From the post-test, it can be concluded that the students were able to answer the problems because they had experience with those types of problems when they were doing the activities. Therefore, we assume that the sequence of activities and also the classroom discussion (the math congress) were greatly influencing the students to progress their learning on multiplication of fractions with natural numbers. More than 60 percents of students showed that after followed the sequence of the activities, they could well perceive the idea of multiplying fractions with natural numbers.

C. Classroom Discussion: New Socio Norms

Interactivity as the fourth tenet of RME emphasizes on students' social interaction to support the individual learning process. In the designed instructional sequence, the explicit negotiation, intervention, discussion, cooperation and evaluation among students and teachers appeared as essential elements in a constructive learning process in which students' informal strategies are used to attain the formal ones. However, in this research the classroom discussion did not run as effective as we had expected. In this classroom, the teacher and the students were still struggling in developing a constructive learning process. This can be caused by teacher's and students' unfamiliarity with the learning process where discussion, negotiations and collaborations play important role. The discussion, negotiations and collaborations connected with constructions and productions are also decisive from bringing the moments of learning. And this is where the educational principle of interaction is applied. However, we found that in this class, the students could be encouraged to follow the instructional activities by giving them assignments which led to free production; by having them, for instance, invent the way how to partition a certain length, perform the representation of the string of yarn and fractions cards hung on it then describe the result.

From the perspective of RME, Elbers and Streefland (Keijzer, 2003) elaborated the classroom settings where the students could share their thinking in the class discussion. In this study, we include the moment in which the students could share their thinking and strategies in solving the problem called *math congress*. However, teacher's role in orchestrating a discussion is not so easy. Only few students were engaged in the discussion while many others were busy doing something else. It can be caused by many students in the class, i.e. 35 students.

To get around of too many students in a class, we tried to group the students into eight small groups (each group consisted of four students). Through this group the students could give more contributions in sharing their thinking. Explaining the personal solutions and listening, then commenting on the partner explanations were also part of the socio norm of the small group activity. The small group activity served as the basis for the whole discussion. The social norms of small group activity became the topics of the math congress.

D. The Role of the Teacher

In this design research, the first role of the teacher started by engaging the students into the problem of '*locating flags and water posts on the running route*' that motivated the students to do measurement (length) activity. The given contextual situation about partitioning the route into eight and six equal parts could be understood by the students.

The way of teacher explaining the situation attracted the students to do the activity. Before the students came up with the idea of partitioning the yarn, they discussed the function of yarn to measure the total length of the route facilitated by the teacher. However, this incident was not as we expected. We expected that the students would realize by themselves that they would need the yarn as helpful tool in partitioning the route. In this case, the teacher directly guided the students to use the yarn. However, she did not lead the students to use the yarn to measure the length of the route. The idea of *measuring* came from the students themselves.

The role of the teacher in this research was not to transmit their knowledge to the students, but to orchestrate the discussion to bring up the mathematical idea in each activity. In this case, teacher who became the actress in this research was not familiar with the learning designed activities in which students more active doing mathematics. However, we could see that this teacher had shown her big effort to create the learning environment even though sometimes she still wanted to tell the answer to the students.

Teacher's role in orchestrating the learning processes also could be seen in the moment of *math congress*. A math congress is held as a space for students to share their ideas. In the math congress, the teacher was successfully functioned as moderator and guided the discussion to bring the students to accept the idea as *taken-as-shared* (Gravemeijer, 1994). The guidance was needed in order to explore the students' logical thinking and reasoning, for instance, in the math congress 1 and 2, the teacher asked some groups to share their thinking by presenting their solution on the board. Teacher's ability in selecting students' solutions was required here. She was successfully choosing the solution which could bring the students to grasp the mathematical idea behind the problems. For instance, in the math congress 2, she brought two different strategies from Singa Mania and Spenta Mania groups where Singa Mania used double number line model while Spenta Mania used their own rule gained from the previous mathematical idea about multiplication of natural numbers by fractions. By comparing and discussing those two strategies, the students could come to the *model for* more formal reasoning in which *general level* of modeling attained by the students.

CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

In conclusion, this study has shown students' learning about multiplication fractions with natural numbers through different levels. The levels are producing fractions, generating equivalencies, operating through mediating quantity, doing one's own productions, and on the way to rules for the operations with fractions. In this study, some ideas and concepts from RME theory has underpinned the design of activities. The context used was about length measurement activity and we found that this is a good context that has allowed students to structure and to mathematize following the five activity levels.

In the first level, students started to produce their own fractions and recalled their knowledge about the meaning of fractions as part of a whole. Starting from a situation, students can cross the border to mathematics on their own, by learning to structure, arrange, symbolize, visualize, and much more. Related to the first tenet of RME, namely *the use of contextual problems*, the length measurement activity serves as the *source* for the mathematics to be produced.

In the second and third levels, students were generating equivalencies and operating through mediating quantity. Problem situation in which the students drew the number line as a representation of string of yarn had a strongly generative nature. Connected to the second tenet of RME, namely *the use of models,* the use of string of yarn here was as a bridge to the number line model which was in more abstract level. At these levels, students could construct and produce their own mathematical knowledge. The use of number line turned into a more complex number line which then called double number line. Finally, the number line was used as a useful model to lead the students to the idea of multiplication of fractions with natural numbers.

The fourth and fifth levels gave opportunity to the students to use their strategies that can be used to develop the next learning process. Connected to the third tenet of RME, namely *the use of students' own creations and contributions*, the representation of string of yarn as a *model of* measuring situation transformed into the number line as a *model for* more formal reasoning. This transformation is another important learning moment for students where they can use the model to move from concrete context to a more formal mathematics.

In this study, we also included *math congress* in which the students could share their thinking and strategies in the third and fifth activities. Connected to the fourth tenet of RME, namely *interactivity,* the classroom settings was elaborated where the students could share their thinking in the class discussion. At this moment, teacher plays an important role in orchestrating the flow of the discussion.

B. Recommendations

Although the study reported here is a relatively small scale study, not all findings of which can be generalized, this study has some remarkable implication for educational practice, especially an important initiative to improve education in Indonesia. Realistic Mathematics Education (RME) can be used as an approach to teach mathematics, or in specific, related to this study, in the topic of multiplication of fractions with natural numbers. It is suggested that if we want to established 'mathematics for all', we should set priorities for all students including the low-achievers. Students who cannot learn formal mathematics should be made to feel welcome, since they have a right to experience mathematics on a level they can understand and use in daily life.

Considering the last tenet in RME, namely *intertwinement*, some activities used in this research could be developed to reach other mathematical topics by intertwining with other mathematics topics. Another mathematics topic that is taught in grade 5 is about *proportion*. We indicated the close relation between proportions and fractions during the learning process. There are also, of course, even more intersections with all kinds of other lines of learning, such as those for division, for practicing the basic operations, measuring, decimal numbers, scale, percentages and probability. Based on observations, it is recommended that these topics be a focus for further studies.

REFERENCES

A. Bakker. (2004). *Design Research in Statistics Education: On Symbolizing and Computer Tools*. Utrecht: CD-β Press, pp. 38-39.

Behr, M. J., Harel, G., Post, T., & Lesh, R. (1992). Rational number, ratio, proportion. In: D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 296-333). New York: Macmillan.

Behr, M. J., Harel, G., Post, T., & Lesh, R. (1993). *Rational numbers: Toward a semantic analysis-emphasis on the operator construct*. In: T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), Rational numbers: an integration of research (pp. 13-47). Hillsdale, NJ: Lawrence Erlbaum.

Cobb, Paul & Gravemeijer, Koeno. (2006). *Educational Design Research*. London & New York: Routledge (Taylor&Francis group).

de Castro, B. (2004). Pre-service teachers' mathematical reasoning as an imperative for codified conceptual pedagogy in Algebra: A case study in teacher education. *Asia Pacific Education Review, 5*(2), 157-166.

Depdiknas. (2006). *Kurikulum Tingkat Satuan Pendidikan Sekolah Dasar*. Jakarta: Depdiknas.

Doorman, L.M. (2005). *Modelling Motion: From Trace Graphs to Instantaneous Change*. Amersfoort: wilco Press.

Fosnot, C.T & Dolk, M. (2002). *Young Mathematicians at Work*. Portsmouth: Heinemann.

Fosnot, C.T & Hellman, L.T. (2007). *Exploring Parks and Playgrounds – Multiplication and Division of Fractions*. Orlando: Harcourt School Publishers.

Freiman, V., & Volkov, A. (2004). *Fractions and fractions again? A comparative analysis of the presentation of common fractions in the textbooks belonging to different didactical tradition.* Paper presented at ICME-10, Discussion Group 14, Copenhagen, Denmark.

Freudenthal, H. (1991). *Revisiting Mathematics Education*. Dordrecht: Kluwer Academic Publisher.

Gravemeijer, K. (1994). *Developing Realistic Mathematics Education* Research Group on Mathematics Education. Utrecht: CD β Press.

Gravemeijer, K & Cobb, P. (2006). Educational Design Research: *Design Research from a Learning Design Perspective* (pp. 45-85). UK: Routledge.

Hiebert, J., & Carpenter, T. P. (1992). *Learning and teaching with understanding*. In: D. A. Grouws (Ed.). Handbook of research on mathematics teaching and learning (pp. 65-97). New York: Macmillan.

Holt, et al. (2003). *Some of the Parts – Mathematics in Context*. United States of America: National Science foundation.

Hunting, R. (1999). Rational-number learning in the early years: what is possible? *Mathematics in the Early Years,* 80-87.

Keijzer, R. (2003). *Teaching Formal Mathematics in Primary Education*. Utrecht: Freudenthal Institute.

Lange Jzn, J. De. (1987). *Mathematics, Insight and Meaning*. Utrecht: OW&OC.

Leinhardt, G. (1988). *Getting to know: tracing students' mathematical knowledge from intuition to competence. Educational Psychologist*, 23, 119-144.

Mack, N. K. (1995). *Confounding whole number and fraction concepts when building on informal knowledge*. Journal for Research in Mathematics Education, 26, 422-441.

Mack, N. (1998). Building a foundation for understanding the multiplication of fractions. *Teaching Children Mathematics, 5*(1), 34-38.

Meagher, M. (2002). Teaching fractions, new methods, new resources. *ERIC Digest.*

Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: a new model and experimental curriculum. *Journal for Research in Mathematics Education, 30*(2), 122 – 147.

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction, 14,* 503-518.

Streefland, L. (1991). *Fractions in realistic mathematics education*. Dordrecht, the Netherlands: Kluwer Academic Publishing.

Streefland, L. (1993). *Fractions: A realistic approach*. In: T. P. Carpenter, E. A. Fennema, & T. A. Romberg (Eds.), Rational numbers an integration of research (pp. 289-325). Hillsdale, NJ: Lawrence Erlbaum.

Taber, S. B. (2002). Go ask Alice about multiplication of fractions. In B. Litwiller & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions: 2002 yearbook* (pp. 61–71). Reston, VA: National Council of Teachers of Mathematics.

Treffers, A. (1987). *Three dimension. A model of Goal and Theory Description in Mathematics Education: The Wiskobas Project*. Dordrecht: Kluwer Academic Publishers.

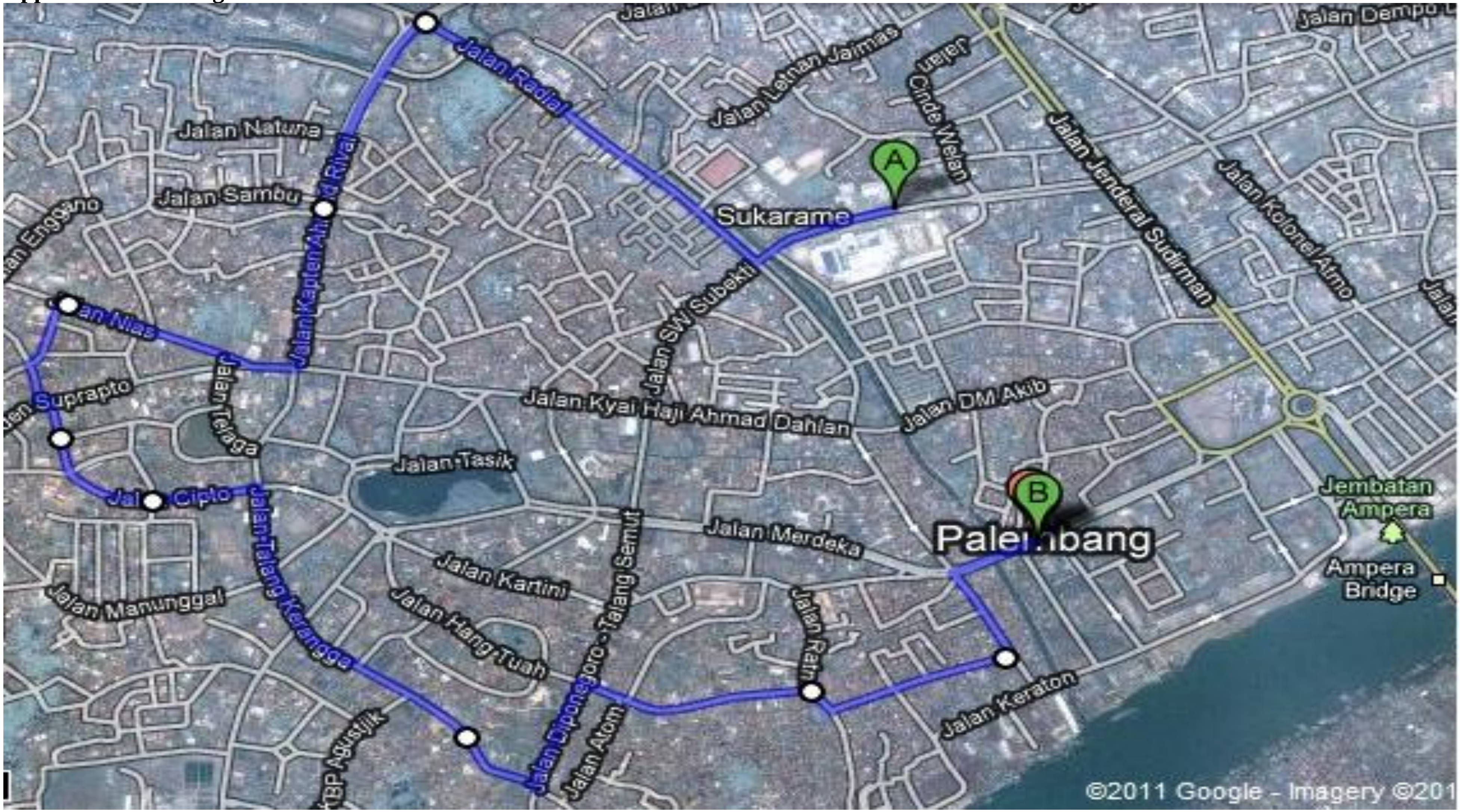
Treffers, A. (1991). Didactical Background of a Mathematics Program voor Primary Education. In: L. Streefland (ed.). *Realistic Mathematics Education in Primary School.* Utrecht: CD-β Press, 21-57.

Tzur, R. (1999). An integrated study of children's construction of improper fractions and the teacher's role in promoting that learning. *Journal for Research in Mathematics Education, 30*(4), 390-416.

Vanhille, L., & Baroody, A. J. (2002). Fraction instruction that fosters multiplicative reasoning. In B. Litwiller & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions: 2002 yearbook* (pp. 224–236). Reston, VA: National Council of Teachers of Mathematics.

Yoshida, H., & Sawano, K. (2002). Overcoming cognitive obstacles in learning fractions: equal-partitioning and equal-whole. *Japanese Psychological Research, 44*(4), 183-195.

Appendix 1: Running Route



APPENDIX 2: PRE-TEST

Name : ____________________________

Date/Classs : ____________________________

PRE-TEST

1. Divide pempek lenjer in the following into:

   a. Four parts equally

How to divide:

   b. Three parts equally

How to divide:

2. Fill in the blank.

   a. $\frac{1}{2} = \frac{....}{8}$ b. $\frac{1}{3} = \frac{4}{....}$

3. Order these fractions from least to greatest .

   a. $\frac{3}{5}, \frac{1}{5}, \frac{5}{5}, \frac{2}{5}, \frac{4}{5}$

   The order of fractions:

   b. $\frac{4}{8}, \frac{1}{8}, \frac{5}{8}, \frac{6}{8}, \frac{3}{8}, \frac{8}{8}, \frac{7}{8}, \frac{2}{8}$

   The order of fractions:

4. Risa buys 6 meters of ribbon. A third of the ribbon is used for wrapping the presents. Determine:

   a. How many meters ribbon used for wrapping the presents?

   My explanation:

b. How many meters the rest of the ribbon? Explain your answer.

<u>My explanation:</u>

APPENDIX 3: POST-TEST

Name : ___________________________

Date/Class : ___________________________

POST-TEST

1. Divide these rope below into 8 and 12 parts equally. Then, write fractions notation for each parts.

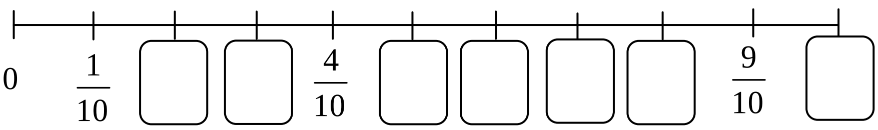
   a. 8 parts equally

   b. 12 parts equally

2. From problem number 1, do you find fractions which are in the same posisition? If yes, what are the fractions? Explain your answer.

My explanation:

3. Complete this fractons' number line.



Explain the relation between fractions $\frac{1}{10}$ and $\frac{5}{10}$!

My explanation:

4. Make you own word problems for these operations with fractions then solve it.

a. $\frac{1}{2} \times 26$

<u>My word problem:</u>

<u>The way to solve:</u>

b. $\frac{1}{4} \times 26$

<u>My word problem:</u>

<u>The way to solve:</u>

c. $\frac{1}{8} \times 26$

<u>My word problem:</u>

<u>The way to solve:</u>

d. $\frac{5}{8} \times 26$

My word problem:

The way to solve:

APPENDIX 4: LESSON PLANS

LESSON PLANS

By: Nenden Octavarulia Shanty

This lesson plans present estimates or projections about what action will be performed at the time of carrying out the learning activities of multiplying fractions with natural numbers.

The students will conduct a series of activities interrelated to each other ranging from producing their own fractions through measurement activities to the formal stage of multiplication of fractions with natural numbers to be passed in stages within six activities as follows.

1. **Locating Flags and Water Posts on the Running Track.**
2. **Notating Fractions in the Empty Cards, Putting the Cards on the String of Yarn, Describing the Relations Among Fractions,**
3. **Math Congress 1,**
4. **Determining Who is Running Farther,**
5. **Math Congress 2, and**
6. ***Minilesson*: Fractions as Multipliers.**

LESSON PLAN

Topics	: Multiplication of Fractions with Natural Numbers
Class	: V
Semester	: II
Activity	: Locating Flags and Water Post on the Running Route
Time Allocation	: 2 × 35 minutes
Meeting	: 1

A. Competency Standard

Using fractions in solving problems.

B. Basic Competence

Multiplying fractions.

C. Indicator

Students are able in partitioning a certain length into several parts which have the same length.

D. Goal

Students make a construction of partitioning, part of a whole.

E. Materials

1. Students' worksheet 1 (one set for each group),
2. The map of running track (printed in A4 size),
3. Yarn (about 1 meter for each group),
4. Scissors (the students are asked to bring it from home, one scissors each group),
5. Two colored markers (each group),
6. Cartoons (one carton each group),
7. Notebook for 'math diary' (each student is asked to prepare a notebook), and
8. Paper glue.

F. Teaching and Learning Activities

1. Pre-Activities (5 minutes)

a. Teacher divides the students into some groups. Students who have abilities above the average will be spread into different groups.

b. Teacher gives student's worksheet 1 and a sheet of carton to each group.

2. Whilst-Activities (30 minutes)

a. After all groups get the materials, teacher gives some time to the students to read questions in this worksheet.

b. Teacher asks one of the students to read aloud questions number 1 and ask other students to read the story in question number 2. Story that needs to be understood by the students is as follows.

To prepare running competition in celebrating Indonesian's Independence Day, Ari and Bimo practice their running skills. They plan to run from Palembang Indah Mall (point A) to Palembang municipality office (point B) following the given running track. Eight flags and six water posts are stored on the track to know the position where Ari and Bimo will stop. Flags and water posts are respectively placed on the running route at the same distance. The last flag and the last water post are stored at the finish line (in front of Palembang municipality office).

c. Teacher discusses together with the students about the story of Bimo and Ari who try to test their running ability.

d. Teacher asks the students to show which one A point and B point in the map then (s)he asks one of the students to come forward to show the running track in the map following the path which coloured by dark blue line. It is aimed to equalize students' perceptions about the route of the track referred to in the story.

e. Teacher asks some students to read aloud questions number 3, 4, 5, and 6. At this moment, teacher equates students' perception about what are asked in the questions.

f. Teacher gives the students some time to solve the problems in this students' worksheet 1.

g. Teacher reminds students the problem that needs to be solved in question number 3. First, the students are asked to determine the location of flags then later the location of water posts. It is aimed to avoid students' misinterpretation which may combine between 8 flags and 6 water posts so that they will divide the running track into 14 parts equally.

h. Teacher observes, motivates, facilitates, and also helps students who need assistance. At this moment, teacher only gives questions which can provoke students' thinking if the students have difficulties. Remind the students to use yarn as a tool to solve the problem. Teacher can also provoke the students with question: *'What is the use of yarn? Relate it with the map of running track!'.*

From this question, it is expected that the students will measure the length of the running track from Palembang Indah Mall (PIM) to Palembang municipality office by mapping the yarn following the map of running track.

Student maps the yarn into the running track.

i. After the students know the length of the running track, conjectures that the students will do are as follows.

- Students will measure the length of the yarn by using ruler to know the length of running track in centimeters.

Student measures the length of the yarn by using ruler.

At this moment, teacher can provoke students' ideas. For instance, the following is a conversation that may occur.

Teacher : What do you do?
Student : I measure the length of the track by using ruler.
Teacher : What is your purpose to do that?
Student : I want to know the total length of the running track.
Teacher : Then what will you do after knowing the total length of the running track.

Student : Because first we need to find the location of flags in the running track, therefore I divide the yarn into 8 parts equally.
Teacher : How do you divide it?
Student : By division number

- Students will fold the yarn. For instance, to find the location of 8 flags, first they may fold the yarn into two, fold it again into two, then the last fold it again into two in such a way that they get 8 parts which the lengths in each part are equal. In this strategy they need to fold the yarn into two parts equally three times. At this moment, their ability in partitioning is required. After that, teacher can provoke students to give mark in the folded parts.

Students folded the yarn then mark the folded parts by using marker.

j. After the students divide the yarn into 8 parts (to know the location of 8 flags) and 6 parts (to know the location of water posts) equally and mark the folded parts by using marker, teacher can provoke students to remap the yarn back to the running route then give signs which represent flags and water posts.
k. After a while, teacher asks to each group to attach their maps which have been marked by the location of flags and water posts in the carton. Cartons will be taped on the wall.

3. Discussion (30 minutes)

a. After all cartons have been taped on the wall, teacher asks each group to observe the work of other groups. After a while, teacher holds a discussion.
b. In this discussion, teacher asks representatives from several groups to share their ideas in locating flags and water posts on the running track in front of the class.
c. Teacher explores students' knowledge about the name of each part in partitioning a certain length into some parts. For instance, what is meant by *a section of eight parts*? From this discussion, it is expected that the students will come up with the idea of fractions which can be discussed in the next meeting.

4. Post-Activities (5 minutes)

a. Teacher asks students to make conclusions in doing this first activity associated with the mathematical ideas.
b. Teacher asks students to write their experience working on the activities today in the math diaries. Completion of this diary can be continued at home.

G. Assessment

Type of assessment : student's work of worksheet 1 (in group) and group presentation

Palembang, March 2011

Teacher,

Researcher,

Sri Horestiati
NIP. 196001181983032007

Nenden Octavarulia Shanty
NIM. 20092512039

Principals of SD Negeri 179 Palembang

Dra. Yuliani, M.M
NIP.195207101978032001

Date/Class : ___________________________

Group : ___________________________

Members' Name : ____

STUDENT'S WORKSHEET 1

1. Look at the map that is given by your teacher.
2. Follow this story when one of your friends read it.

To prepare running competition in the celebration of Indonesian Independence Day, Ari and Bimo practice their running skills. They plan to run from Palembang Indah Mall (point A) to Palembang district office (point B) following the running route. Eight flags and 6 water posts are stored on the track to know the position where Ari and Bimo will stop. Flags are placed on the running route with the same distance. The water posts also are placed on the running route with the same distance. The last flag and the last water post are stored at the finish line (in front of Palembang district office).

3. From the story above, determine:
   a. The location of 8 flags, with the same distance among the flags.
   b. The location of 6 water posts, with the same distance among the water posts.
4. Give marks on the route showing the location of the flags and the water posts.
5. You are allowed to use materials (yarn, scissors, markers) that are provided if it is needed.
6. Explain your way to determine the location of the flags and the water posts in the box below.

My explanation:


Jalan Radial
Jalan Letnan Jaimas
Jalan Cinde Welan
Jalan Dempo
Jalan Natuna
Jalan Sambu
Jalan Kapten Ahmad Rivai
Sukarame
A
Jalan Jenderal Sudirman
Jalan Kolonel Atmo
Jalan Enggano
Jalan SW Subekti
Jalan Nias
Jalan Teraga
Jalan Kyai Haji Ahmad Dahlan
Jalan DM Akib
Suprapto
Jalan Tasik
Cipto
Jalan Talang Kerangga
B
Palembang
Jembatan Ampera
Ampera Bridge
Jalan Merdeka
Jalan Kartini
Jalan Manunggal
Jalan Hang Tuah
Jalan Diponegoro - Talang Semut
Jalan Ratu
Jalan Keraton
Jalan Atom
KBP Agustjik
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LESSON PLAN

Topic	: Multiplication of Fractions with Natural Numbers
Class	: V
Semester	: II
Activity	: Notating Fractions in the Empty Cards, Putting the Cards on the String of Yarn, and Describing the Relations among Fractions
Time Allocation	: 2 × 35 minutes
Meeting	: 2

A. Competency Standard

Using fractions in solving problems.

B. Basic Competence

Multiplying fractions.

C. Indicators

1. Students are able to symbolize fractions as a result of partitioning certain length into several parts.
2. Students are able to discover the relation among fractions.

D. Goals

1. Students will symbolize the result of partitioning and show it on the string of yarn.
2. Students will describe the relations among fractions such as equivalence of fractions.

E. Materials

1. Students' woksheet 2,
2. Whiteboard,
3. Markers,
4. Empty fractions cards (each group gets one set of 14 empty cards which consists of 8 red cards and 6 blue cards),
5. Yarn that have been used to locate flags and water posts in the first meeting,
6. Scissors, and
7. Cellophane tape.

F. Teaching and Learning Activities

1. Pre-Activities (5 minutes)

a. Teacher groups students with the same member as in the first activity.

b. Teacher gives student's worksheet 2 and one set of 14 empty cards to each group.
c. Teacher explains that this activity is associated with activity in the previous meeting. Invite a few students to recall the activity they have done in the previous meeting.
d. Teacher reminds the students to reuse the yarn that have been used in the first meeting which has been marked by using marker to locate flags and water posts.

2. Whilst-Activities (30 minutes)

a. Teacher asks one of the students to read aloud questions number 1 and 2.
b. In symbolizing fractions notation from the result of partitioning, students might have difficulties. Teacher can provoke the students by posing question: *'If a yarn is divided into eight parts, what is the value of each part?'*
c. When notating fractions, probably there are some students who still use unit fractions (for instance, give a name $\frac{1}{8}$ for every part when partitioning the track into eight parts equally) instead of non unit fractions (for instance, $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, etc).
d. Students hang the fractions cards on the yarn that they used in the first activity which have been marked by the position of flags and water posts.

Students try to put fractions cards on the yarn.

e. Teacher gives the students some time to solve the problems in this students' worksheet 2.

f. Teacher observes, motivates, facilitates, and also helps students who need assistance. Students might have difficulties in working out problem number 4, when they are asked to describe their findings from the drawing of yarn and the fractions cards. Teacher can provoke students by posing question: *'What do you find from your drawing in number 3? Note the position of the fractions.'*

g. Furthermore, students might have difficulties in understanding problem number 5 because the words $\frac{1}{8}$*-jump*' somewhat difficult to understand by the students. Teacher can provoke the students by posing question: *'From what point to what point does it jump? If you jump* $\frac{1}{8}$ *each jump and you jump from zero point to* $\frac{5}{8}$*, how many jumps have you going through?'.*

h. In question number 6, students are asked to remember the concept of multiplication in natural numbers as repeated addition. This part is very important which will underlies students' conclusion when they relate repeated addition in natural numbers with repeated addition in fractions in question number 7.

i. Teacher asks the students to put their fraction cards which have been hung on the string of yarn on their poster.

Figure 5. Students' work of first activity and fractions cards which hang on the yarn.

3. Discussion (25 minutes)

a. After a while, teacher holds class discussion.

b. Teacher discusses the issues contained in this worksheet.

c. Teacher asks students' findings from their work of puting fractions cards on the string of yarn. If there is a students or even more who find fractions in the same position, for instance $\frac{3}{6}$ and $\frac{4}{8}$ or $\frac{6}{6}$ and $\frac{8}{8}$, teacher can challenge the students to explain why it can be the same. Teacher provokes students to come up with the idea of $\frac{3}{6}=\frac{4}{8}$ which later on will go to the concepts of equivalent fractions and simplifying fractions. The mathematical idea is if two fractions are equivalent, the ratio between the numerators is the same as the ratio between the denominators – to maintain equivalence, the ratio of the related number must be kept constant.

d. Teacher provokes students to share the idea of *jumps* in the question number 5.

e. Teacher discusses about the concept of multiplication of natural numbers such as given in the question number 6 but using different number. For instance, *'what is meant by multiplication of* 6×8 *? 8 times of 6 or 6 times of 8?'*.

f. Teacher provokes students to link the idea of multiplication of natural numbers by fractions as repeated addition of fractions. For instance:

$$\frac{5}{8}=5\times\frac{1}{8}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$$

4. Post-Activities (10 minutes)

a. Students make conclusions about equivalent fractions which shown by fractions in the same position, simplifying fractions, and the concept of multiplication of natural numbers by fractions as repeated addition of fractions.

b. Teacher asks students to write their experience working on the activities today in the math diaries. Completion of this diary can be continued at home.

G. Assessment

Type of assessment : Student's work (in group)

Palembang, March 2011

Teacher, Researcher,

Sri Horestiati Nenden Octavarulia Shanty

NIP. 196001181983032007 NIM. 20092512039

Principals of SD Negeri 179 Palembang

Dra. Yuliani, M.M.

NIP. 195207101978032001

Date/Class : ___________________________

Group : ___________________________

Members' Name : ____

STUDENT'S WORKSHEET 2

1. From the results of locating the flags and the water posts on the track (Activity 1), write down fractions on fractions card for each part of the running track.
2. Hang your fractions cards on the yarn (note: if you use yarn in the first activity, you can use it again).
3. Draw your streched yarn and the fractions cards.

My drawing:

4. What do you find? Explain your findings in the box below.

My findings:

5. Look at again your answer in number 3. How many $\frac{1}{8}$ jumps from starting point to $\frac{5}{8}$? Explain your answer.

My explanation:

6. What do you know about 2×3? Explain your answer.

My explanation:

7. Explain the relation between $\frac{1}{8}$ jumps with point $\frac{5}{8}$!

My explanation:

LESSON PLAN

Topic	: Multiplicaton of Fractions with Natural Numbers
Class	: V
Semester	: II
Activity	: Math Congress 1
Time Allocation	: 2 × 35 minutes
Meeting	: 3

A. Competency Standard

Using fractions in solving problems.

B. Basic Competence

Multiplying fractions.

C. Indicators

1. Students are able to understand the meaning of fractions within measurement activity which has been done in the first meeting.
2. Students are able to write fractions notation from the result of partitioning certain length into several parts with the same length.
3. Students are able to find the concept of equivalent fractions which indicate by fractions in the same position.
4. Students are able to explain the relation among fractions.

D. Goals

1. Students will share their ideas and their experiences in partitioning the track, symbolizing the result of partitioning, and the relations among fractions i.e. equivalent fractions.
2. Students will construct multiplicative reasoning within equivalent fractions.

E. Materials

1. Student's worksheet 3,
2. Poster (students' works and posters from activity 1 and 2),
3. Whiteboard, and
4. Marker.

F. Teaching and Leraning Activities

1. Pre-Activities (5 minutes)

a. Teacher opens the learning process by asking the students about what they have done in the previous meeting.

b. Teacher gives student's worksheet 3 to each students.

2. Whilst-Activities (50 minutes)

a. Teacher holds class discussion.

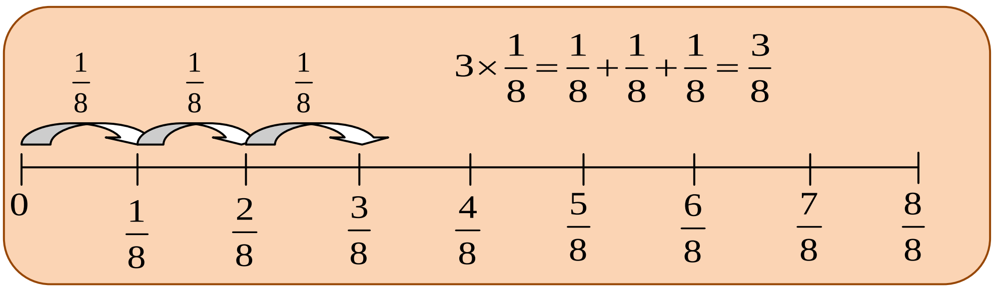
b. Teacher can discuss about the strategies of the students in solving problem in activity in Day One and Two which will provide a discussion of several big ideas such as:

- Fractions can be considered as measurement. It is obvious when students try to find the total length of the track, they do the measurement activity.
- Making fractions means dividing something into some parts which is equal.
- Notation of fractions.

  The fractions are fractions produced by the students from partitioning the running track into eight and six parts (activity 1). The fractions can be unit fractions (e.g., $\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6}$ and $\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8},\frac{1}{8}$) and non unit fractions (e.g., $\frac{1}{6},\frac{2}{6},\frac{3}{6},\frac{4}{6},\frac{5}{6},\frac{6}{6}$ and $\frac{1}{8},\frac{2}{8},\frac{3}{8},\frac{4}{8},\frac{5}{8},\frac{6}{8},\frac{7}{8},\frac{8}{8}$).
- Number line of fractions

  Teacher discusses the drawing represented the yarn with fractions hanging on it. If there is a group who draw a line as a represntation of a yarn, then bring this findings into the discussion. It is expected that the students will come up with the idea of number line of fractions.
- Equivalent fractions

  From students findings about fractions in the same position, teacher can discuss about equivalent fractions such as $\frac{4}{8}$ and $\frac{3}{6}$ also $\frac{8}{8}$ and $\frac{6}{6}$.
- Introducing the concept of multiplication as repeated addition using a familiar word *jumps*.



To strengthen the use of double number line, teacher can give other examples. For instance, teacher can ask the students '*what is the relation between* $\frac{1}{8}$ *and* $\frac{7}{8}$ *?, how many* $\frac{1}{8}$*-jumps from zero point (0) to* $\frac{7}{8}$ *?'*

3. Post-Activities (15 minutes)

a. Teacher asks the students to fill in students' worksheet 3 about the matters that have been discussed in this math congress.

b. Teacher asks the students to write their experiences in working on the activities today in the math diaries. Completion of this diary can be continued at home.

G. Assessment

Type of assessment: student's work (individual)

Palembang, March 2011

Teacher, Researcher,

Sri Horestiati
NIP. 196001181983032007

Nenden Octavarulia Shanty
NIM. 20092512039

Principals of SD Negeri 179 Palembang

Dra. Yuliani, M.M.
NIP. 195207101978032001

Date/Class : ___________________________

Name : ___________________________

<u>STUDENT'S WORKSHEET 3</u>

MATH CONGRESS 1

1. Discuss together with your friends your experiences and the ways you solve the problems in Activity 1 and 2.

2. Write down your result of the discussion in the following box.

The result of discussion:

LESSON PLAN

Topic	: Multiplication of Fractions with Natural Numbers
Class	: V
Semester	: II
Activity	: Determining Who is Running Farther
Time Allocation	: 2 × 35 minutes
Meeting	: 4

A. Competency Standard

Using fractions in solving problems.

B. Basic Competence

Multiplying fractions.

C. Indicators

1. Students are able to compare fractions.
2. Students are able to multiply fractions with natural numbers.

D. Goals

1. Students will compare fractions within a certain length.
2. Students informally use fractions as multipliers.

E. Materials

1. Student's worksheet 4,
2. Whiteboard, and
3. Markers.

F. Teaching and Learning Activities

1. Pre-Activities (10 minutes)

a. Teacher opens the learning activity by asking the students to remember the story about Ari and Bimo who have running practice in Activity 1.

b. Teacher gives student's worksheet 4 to each group.

2. Whilst-Activities (55 minutes)

a. Teacher gives some time to the students to read question in this worksheet.

b. After that, teacher asks one of the students to read aloud the story in question number 1.

After all flags and water posts are in its position, Ari and Bimo start their training. They know the track length from Palembang Indah Mall to Palembang municipality office is 6 kilometers. After running for a while, *Bimo decides to stop because he is exhausted. He stop at the fifth flag. Ari also decides to stop at the fourth water post.*

c. Teacher holds an initial discussion before the students work on this problem. At this moment, teacher should make sure that all students could understand the situation and what is the problem that needs to be solved.

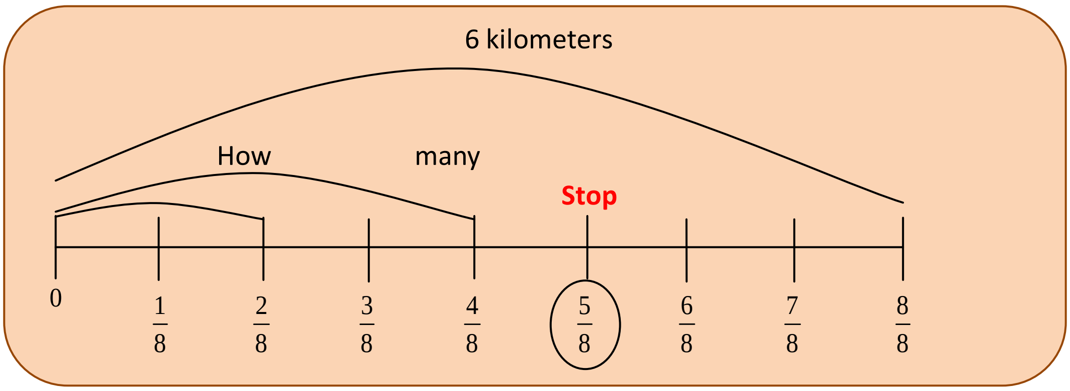
d. Teacher gives attention to the something which is important in the story. In this case the sentences: '*Bimo stops in the fifth sign. Ari stops in the fourth water post'* and also the length of the running race route (6 kilometers) plays an important role to solve this problem.

e. Teacher reminds the students about the number of flags and water posts on the running track.

f. Teacher provokes the students to come to the idea of *five of eighth* and *four of sixth* and how they present it in the fractions notation.

g. Teacher gives the students some time to solve the problems in this students' worksheet 4.

h. Teacher observes, motivates, facilitates, and also helps students who need assistance. At this moment teacher could only poses questions which can provoke students' thinking when they have difficulties. Teacher can also introduce double number line model to the students. Teacher only gives a short clue then let the students develop and continue next idea. For instance, teacher can draw double number line as follows.



From double number line above, teacher can provoke students by posing question: '*If it is known the length of the running track is 6 kilometers, how many kilometers a half?*

3. Post-Activities (5 minutes)

a. Teacher asks students to make conclusions in doing this fourth activity associated with the mathematical ideas.
b. Teacher asks students to write their experience working on the activities today in the math diaries. Completion of this diary can be continued at home.

G. Assessment

Type of assessment: student's work (in group).

Palembang, March 2011

Teacher, Researcher,

Sri Horestiati Nenden Octavarulia Shanty

NIP. 196001181983032007 NIM. 20092512039

Principals of SD Negeri 179 Palembang

Dra. Yuliani, M.M.

NIP. 195207101978032001

Date/Class : ___________________________

Group : ___________________________

Members' Name : ____

STUDENTS' WORKSHEET 4

1. Read the following story carefully. This story is still related to the story in the first activity.

   After all flags and water posts are in its position, Ari and Bimo start their training. They know the track length from Palembang Indah Mall to Palembang municipality office is 6 kilometers. After running for a while, Bimo decides to stop because he is exhausted. He stop at the fifth flag. Ari also decides to stop at the fourth water post.

2. *How many kilometers have Bimo and Ari run*? Explain your answer.

My explanation:

3. Who is running farther? Why?

<u>My explanation:</u>

LESSON PLAN

Topic	: Multiplication of Fractions with Natural Numbers
Class	: V
Semester	: II
Activity	: Math Congress 2
Time Allocation	: 2×35 minutes
Meeting	: 5

A. Competency Standard

Using fractions in solving problems.

B. Basic Competence

Multiplying fractions.

C. Indicators

1. Students are able to multiply fractions with natural numbers.
2. Students will use the idea of multiplication of fractions with natural numbers in solving problems.

D. Goals

1. Students will share their ideas and their experiences in informally using fractions as multipliers.
2. Students will discuss several big ideas which will appear in the discussion.

E. Materials

1. Student's worksheet 5,
2. Students' works of activity 4,
3. Whiteboard, and
4. Marker.

F. Teaching and Learning Activity

1. Pre-Activities (5 minutes)

a. Teacher opens the learning process by asking the students to remind activity 4.
b. Teacher gives student's worksheet 5 to each student.

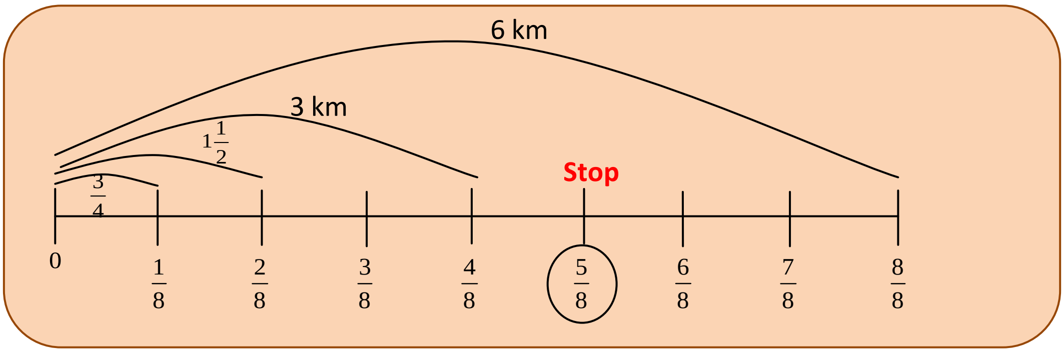
2. Whilst-Activities (55 minutes)

a. Teacher gives opportunity to several groups to explain their strategies in solving problem in student's worksheet 4.

b. After the students share their experiences, teacher holds class discussion. The variety of strategies from students answer from the previous activity is likely to provide a discussion – the big ideas as follows.

- Fraction can be decomposed into unit fractions or 'friendly' fractions.
- Fractions can be considered as multipliers on other numbers. For instance, when students need to find '*how many kilometers that has been taken by Bimo and Ari?'*. They need to find $\frac{5}{8}$ of 6 kilometers and $\frac{4}{6}$ of 6 kilometers.
- The distributive property holds for multiplication over addition for fractions. Partial products ($\frac{1}{2}\times 6$ and $\frac{1}{8}\times 6$) can be used to determine the product of $\frac{5}{8}\times 6$.
- Using proportional reasoning, students may determine that if $\frac{1}{2}$ of 6 kilometers is 3 kilometers, then $\frac{1}{4}$ of 6 is $1\frac{1}{2}$ kilometers, and $\frac{1}{8}$ of 6 is $\frac{3}{4}$ kilometers.

c. After a while, if there are no anwers from students which will lead to the double number line model where fractions and the length of the running track located on the number line, the teacher can introduce the use of the double number line as one strategy in solving the problem. Teacher does not introduce the whole process in solving the problem using double number line model. For instance, teacher only provokes the students by asking their strategies in partitioning the running track into eight parts equally.



From the double number line model, there are some possibilities of students' answer in determining $\frac{5}{8}$ of 6 kilometers as follows.

- If the students relate the double number line with the word '*jumps*', they will add $\frac{3}{4}$ five times. It is because there are five jumps from zero point to $\frac{5}{8}$. Example of the calculation is as follows.

  $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=\frac{15}{4}$

  From repeated addition of fractions, students can relate this into multiplication of natural number with fractions which can be written as $5\times\frac{3}{4}$. At this moment teacher can provoke the students to change fractions into mixed fraction so that $\frac{15}{4}$ will produce mixed fractions $3\frac{3}{4}$.

- Students only add 3 kilometers with $\frac{3}{4}$ kilometers. It s because to reach $\frac{5}{8}$, it has been taken half way which is 3 kilometers then they just need to add one jump more which the length was $\frac{3}{4}$ kilometers.

d. Teacher gives students some time to explore the use of double number line to solve problem in students' worksheet 4.
e. Teacher gives another example which still related with problem in students' worksheet 4. For instance, teacher changes the number of the flags become 12 flags and the length of the track become 24 kilometers. Ask the students to find the distance from zero point to $\frac{7}{12}$ by using double number line model.

3. Post-Activities (10 minutes)

a. Teacher asks the students to fill in students' worksheet 5 about the matters that have been discussed in this Math Congress 1.
b. Teacher asks students to write their experience in working on the activities today in the math diaries. Completion of this diary can be continued at home.

G. Assessment

Type of assessment: student's work of worksheet 5 (individual).

Palembang,   March 2011

Teacher,	Researcher,
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Principals of SD Negeri 179 Palembang

Dra. Yuliani, M.M

NIP. 195207101978032001

Date/Class : ___________________________

Name : ___________________________

STUDENTS' WORKSHEET 5

MATH CONGRESS 2

1. Discuss together with your friends your experiences and the ways you solve the problems in Activity 4.
2. Write down your result of the discussion in the following box.

The result of discussion:

LESSON PLAN

Topic	: Multiplication of Fraction with Natural Numbers
Class	: V
Semester	: II
Activity	: Minilesson*:* Fractions as Operator
Time Allocation	: 2 × 35 minutes
Meeting	: 6

A. Competency Standard

Using fractions in solving problem.

B. Basic Competence

Multiplying fractions.

C. Indicator

Students are able to solve multiplication of fractions with natural numbers.

D. Goals

1. Students are able to make their own word problem from the string of number in this minilesson.
2. Students are given opportunity to use their strategies in solving the problems from the previous activity.

E. Materials

Student’s worksheet 6, whiteboard, and marker.

F. Teaching and Learning Activities

1. Pre-Activities (5 minutes)

a. Teacher opens the learning process by reviewing the matters that have been discussed in the Math Congress 2.

b. Teacher gives students’ worksheet 6 to each group.

2. Whilst-Activities (60 minutes)

a. Teacher gives some time to the students to solve all problems in this students’ worksheet 6.

b. Teacher observes, motivates, facilitates, and also helps students who need assistance.

c. After the students finish all the problems in this worksheet, the teacher discusses about the big ideas and strategies in solving the problem.

3. **Closing (5 minutes)**

   Teacher asks students to write their experience working on the activities today in the math diaries. Completion of this diary can be continued at home.

G. Assessment

Type of assessment : student's work of worksheet 6 (individual)

Palembang, March 2011

Teacher,	Researcher,
Sri Horestiati	Nenden Octavarulia Shanty
NIP. 196001181983032007	NIM. 20092512039

Principals of SD Negeri 179 Palembang

Dra. Yuliani, M.M

NIP. 195207101978032001

Date/Class : ____________________________

Name : ____________________________

STUDENT'S WORKSHEET 6

Make your own word problems from these multiplications of fractions with natural numbers then solve it.

1. $\frac{1}{2} \times 34$

My word problem

The way I solve the problem

2. $\frac{1}{4} \times 34$

My word problem

The way I solve the problem

3. $\frac{1}{8}\times 34$

My word problem

The way I solve the problem

4. $\frac{5}{8}\times 34$

My word problem

The way I solve the problem

APPENDIX 5: TABLE OF ANALYSIS OF POST-ASSESSMENT

Question Number	Mathematical Ideas	Analysis
1.	Fractions represent a relation: making partition means dividing certain length into some parts.	Partitioning rope into 8 parts equally Twenty six students could partition the rope into eight parts equally. Some used ruler to measure the length of the rope then divide the length by 8. Others use the strategy of dividing the rope by firstly finding the middle point of the rope (dividing the rope into two first). Most of students who partition incorrectly used the strategy of left to right partition. They estimated the length of each partition and continued to partition till got 8 parts. However, the last point of partition was not at the last point of the rope. They probably had not perceived the idea of partitioning a certain length (from starting point to the last point). 26 students = 81,25 %
		Partitioning rope into 12 parts equally Twenty two students could partition the rope into twelfth parts equally. They used the same strategy as in problem 1a. Students who could not partition the rope into 12 parts equally had problem in the part of partition the folded part into three. This point where many students made mistakes. They also used the strategy of left to right partition. 22 students = 68,75 %
2.	Fractions that are in the same position are equal fractions.	Twenty students could answer correctly. They could find there are four pairs of fractions which are in the same position (i.e., $\frac{2}{8}$ and $\frac{3}{12}$, $\frac{4}{8}$ and $\frac{6}{12}$, $\frac{6}{8}$ and $\frac{9}{12}$, $\frac{8}{8}$ and $\frac{12}{12}$). However, there were some students who partitioned the rope in the first problem correctly, but they only find two or three pairs of fractions. This moment showed students' level of foresight which is required in this problem. 20 students = 62,5 %
3.	Multiplication by fractions as a repeated addition of fractions can be introduced within equivalent fractions.	All students could complete fractions in the number line of fractions because it arranged from least to greatest fractions and there were some fractions as a clue. Twenty eight students could answer that there were five jumps of $\frac{1}{10}$-jumps from zero point to $\frac{5}{10}$. But only 21 students could describe the relation between $\frac{1}{10}$-jumps and $\frac{5}{10}$ to come up with the idea of $\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}=\frac{5}{10}$, add $\frac{1}{10}$ as much as 5 times $5\times\frac{1}{10}$. 21 students = 65,63 %

4.	Multiplying fractions by natural numbers in which fractions function as multipliers.	Making words problem Fifteen students could nicely make word problem of multiplication of fractions by natural numbers problem. Most of them still related it with running problem (as in problem 4 in the activity). Other students still had difficulties in making word problem and distinguishing, for instance, the point of $\frac{1}{4}$ with $\frac{1}{4}$ kilometers. They also often forgot to include the distance of the running route to represent the natural numbers. 15 students = 46,88 %
		Solving multiplication of fractions by natural numbers problem Twenty one students could solve correctly the string of number in problem 4. Most of them used Dhea's idea about the rule of operation with fractions because they thought this ways was the most efficient strategy. Two students who also answer all the string of numbers correctly used double number line model as a helpful tool. 21 students = 65,63 %

CURRICULUM VITAE

The writer of this thesis, Nenden Octavarulia Shanty, was born in Jakarta, October 30, 1984. Taking the first formal education in kindergarten Cenderawasih East Bekasi graduated in 1991. Continuing the study to elementary school SDN I East Bekasi graduated in 1996. After that went to junior high school SMPN I Purwakarta West Java graduated in 1999. Then proceed to high school SMUN I Purwakarta graduated in 2002. In 2002 studied at the Faculty of Mathematics and Natural Sciences Mathematics Department, Mathematics Education Studies Program, State University of Jakarta graduated in 2007. She then follows International Master Program on Mathematics Education in collaboration between Sriwijaya University and Utrecht University start from August 2009.

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